ATOMS

BOHR MODEL OF THE HYDROGEN ATOM

BOHR'S MODEL OF AN ATOM

Between 1913 and 1915, Niels Bohr developed a quantitative atomic model for the Hydrogen atom, aiming to explain its spectrum. This model integrated the nuclear model of the atom proposed by Rutherford, based on his experimental findings. The Bohr model demonstrated success in predicting the essential features of the spectrum emitted by the Hydrogen atom during this period. It was specifically tailored for Hydrogenic atoms, which consist of a nucleus with a positive charge of +Ze (where Z is the atomic number, and e is the charge of the electron) and a single electron. The Bohr model, however, did not account for more complex electron-electron interactions within an atom, limiting its validity to one-electron systems or Hydrogenic atoms.

The Bohr model found applicability in describing one-electron systems like H, He+, Li+2, etc., providing reasonable explanations for the observed features in the spectrum emitted by such Hydrogenic atoms. Nevertheless, it fell short of presenting a completely accurate representation, as the true nature of these simple atoms involves quantum mechanics, which differs fundamentally from Bohr's model. Due to its incorporation of classical and modern physics elements, the Bohr model is now termed a semi classical model. Bohr elucidated his atomic model through three postulates, each contributing to its overall understanding. Let's delve into these postulates one by one.

PROPERTIES OF ELECTRON IN BOHR'S ATOMIC MODEL

Now, let's explore the fundamental characteristics of an electron as it revolves in stable orbits, referred to as Bohr energy levels. As previously discussed, certain orbits meet the conditions outlined in the first and second postulates of the Bohr model, making them stable for electron revolution around the nucleus. These stable orbits correspond to specific quantum numbers, denoted as n = 1, 2, 3, and so on. For a given nth orbit, assuming its radius is represented by r_n, and the electron is in motion with a speed v_n, various physical parameters associated with the electron in the nth orbit can be expressed using a subscript n with the corresponding symbols, such as r_n,v_n etc.

(a) Radius of nth Orbit in Bohr Model

The calculation of the radius of the electron in the nth Bohr's orbit involves applying the first two postulates of the Bohr model, utilizing the equations discussed earlier. The expression for the radius (r_n) is derived as follows:

$$V_n = \frac{nh}{2\pi m r_n}$$

Inserting this value of v_n into the equation $mvr = \frac{nh}{2\pi}$, *we get*

$$r_n = \frac{n^2h^2}{4\pi^2 KZe^2 m}$$

$$r_n = \frac{h^2}{4\pi^2 Ke^2 m} \times \frac{n^2}{Z}$$

$$r_n = 0.529 \times \frac{n^2}{Z} A$$

2 Velocity of Electron in the nth Bohr's Orbit By substituting the value of r_n, we can calculate the velocity v_n as

$$v_n = \frac{2\pi KZe^2}{nh}$$

$$v_n = \frac{2\pi Ke^2}{h} \times \frac{Z}{n}$$

$$v_n = 2.18 \times 10^6 \times \frac{Z}{n} m/s$$

(b) Time period of Electron in n^{th} Bohr's Orbit

Time period of the electron in the n^{th} orbit is given by

$$T_n = \frac{1}{f_n} \text{ Or } T_n = \frac{n^3h^3}{4\pi^2 K^2 Z^2 e^4 m}$$

(c) Current in nth Bohr's Orbit

When electrons revolve around the nucleus in the nth Bohr's Orbit, the revolution generates a current in the orbit. According to the definition of current, the current in the nth orbit is the total coulombs passing through a point in one second. In an orbit, an electron passes through a point f_n times in one second. Therefore, the current in the nth orbit will be

$$I_n = f_n \times e \text{ Or } I_n = \frac{4\pi^2 K^2 Z^2 e^5 m}{n^3h^3}$$

(d) Energy of Electron in n^{th} Orbit

We have established that in the nth orbit, during revolution, the total energy of the electron can be expressed as the sum of the kinetic and potential energy of the electron, represented as:

$$E_n = K_n + U_n$$

The kinetic energy of an electron in the n^{th} orbit can be expressed as:

$$K_n = \frac{1}{2}mv_n^2$$

From the equation derived from the first postulate of the Bohr Model for the nth orbit, we obtain:

$$mv_n^2 = \frac{KZe^2}{r_n}$$

From equation

$$K_n = \frac{1}{2}mv_n^2 = \frac{1}{2}\frac{KZe^2}{r_n}$$

The potential energy of an electron in the n^{th} orbit is expressed as follows:

$$U_n = -\frac{KZe^2}{r_n}$$

Hence, the total energy of an electron in the nth orbit can be expressed as:

$$E_n = K_n + U_R = \frac{1}{2}\frac{KZe^2}{r_n} - \frac{KZe^2}{r_n} = \frac{1}{2}\frac{KZe^2}{r_n}$$

Here we can see that $|E_n| = |K_n| = \left|\frac{1}{2}\right| |U_n|$ which a very useful relation, always followed by is

Substituting the value of r_n into the expression, which pertains to a particle undergoing circular motion subject to the influence of a force following an inverse square law, yields the following result:

$$E_n = -\frac{1}{2}KZe^2 \times \frac{4\pi^2 KZe^2 m}{n^2h^2}$$

$$= -\frac{2\pi^2 K^2 Z^2 e^4 m}{n^2 h^2}$$

$$E_n = \frac{2\pi^2 K^2 Z^2 e^4 m}{h^2} \times \frac{Z^2}{h^2}$$

By substituting the values of the constants into the above equation, we obtain the following result:

$$E_n = -13.6 \times \frac{Z^2}{n^2} eV$$

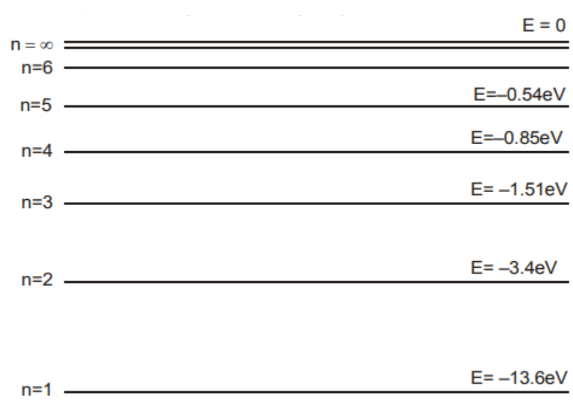
The equation above can be utilized to determine the energies of electrons in various energy levels of different hydrogen atoms.

(e) Energies of different energy level in Hydrogenic atoms

Using the above equation, one can calculate the energies of different energy levels. It is essential for students to memorize the energies for the first six levels as follows:

$E1 = -13.6\ Z^2$ eV
$E2 = -3.40\ Z^2$ eV
$E3 = -1.51\ Z^2$ eV
$E4 = -0.85\ Z^2$ eV
$E5 = -0.54\ Z^2$ eV
$E6 = -0.36\ Z^2$ eV

The equations above clearly demonstrate that as the value of n increases, the difference between two consecutive energy levels decreases. This relationship is visually represented in the energy level diagram for a hydrogen atom, as depicted in the accompanying figure.



Now, if we multiply both the numerator and denominator of the above equation by *ch*, we obtain:

$$E_n = \frac{2\pi^2 K^2 e^4 m}{ch^3} \times ch \times \frac{Z^2}{n^2}$$

$$E_n = -Rch \times \frac{Z^2}{n^2} eV$$

Here $R=\dfrac{2\pi^2K^2e^4m}{ch^3}$ is defined as the Rydberg Constant, with a value of $R = 10967800\ m^{-1}$, which can be approximated as $10^7 m^{-1}$. For $n = 1$ and $Z = 1$, the energy is expressed as:

$E = -R_{ch}$ joules and is called as One Rydberg Energy

1 Rydberg $= 13.6\ eV = 2.17 \times 10^{-18}$ joules

Certainly, let's explore a few examples of Bohr's atomic model to enhance our understanding.

Example.

What is the angular momentum of an electron in Bohr's Hydrogen atom if its energy is -3.4 eV?

Solution.

The energy of an electron in the nth Bohr orbit of a hydrogen atom is expressed as follows:

$$E=\frac{13.6}{n^2}eV$$

$$-3.4=-\frac{13.6}{n^2}$$

$$n^2 = 4$$

$$n = 2$$

The angular momentum of an electron in the nth orbit is determined by the equation $L=\dfrac{nh}{2\pi}$. Substituting $n = 2$ into the equation, we get:

$$L=\frac{2h}{2\pi}=\frac{h}{\pi} \Rightarrow T=\frac{n^3h^3}{4\pi^2K^2Z^2e^4m}$$

$$T \alpha n^3 \quad or \quad \frac{T_1}{T_2}=\frac{n_1^3}{n_2^3}$$

Given that $T_1 = 8T_2$, the relation mentioned above can be expressed as:

$$\left(\frac{n_1}{n_2}\right)^3=8 \quad or \quad n_1=2n_2$$

Therefore, the potential values for n_1 and n_2 are:

$n_1 = 2, n_2 = 1;\ n_1 = 4, n_2 = 2;\ n_1 = 6, n_2 = 3$ And so on.........