WAVE OPTICS

REFRACTION AND REFLECTION OF PLANE WAVES USING HUYGENS PRINCIPLE

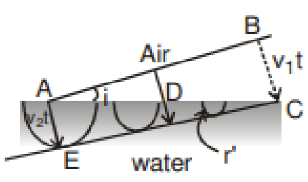
REFLECTION AND REFRACTION:

We can employ a modified version of Huygens' construction to gain insight into the reflection and refraction of light. In Figure (a), we observe an incident wave front at an angle θ with the surface that separates two different media, such as air and water. The phase speeds in these two media are represented as v_1 and v_2. At the moment when point A on the incident wave front reaches the surface, point B is still on its way and has to traverse the distance BC, which equals AC sin(θ). This journey takes a specific amount of time, t, which is determined by $t = BC/v_1 = AC\ (\sin\theta)/\ v_2$. After this time interval, a secondary wave front with a radius of v_2t and center at point A would have entered medium 2. Simultaneously, a secondary wavelet originating from a point D located between A and C would have a radius smaller than v_2t. In contrast, the wave front in medium 2 takes the form of a line passing through point C and tangential to the circle centered at point A.

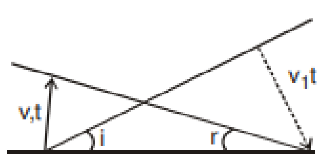
This configuration allows us to determine the angle r′, which characterizes the refracted wave front with respect to the surface. It is given by $AE = v_2t = AC\sin r'$, and subsequently, $t = AC\ (\sin r')/v_2$. By equating these two expressions for t, we obtain the law of refraction in the form $\sin\theta/\sin r' = v_1/v_2$. A similar illustration is presented in Figure (b) to depict the reflected wave, which travels back into medium.

In this scenario, we denote the angle formed by the reflected wave front with the surface as 'r,' and interestingly, we find that 'i' is equal to 'r.' It's worth noting that in both cases of reflection and refraction, we observe secondary wavelets originating at different starting times. This is in contrast to the previous application (as shown in the figure) where we initiated them simultaneously.

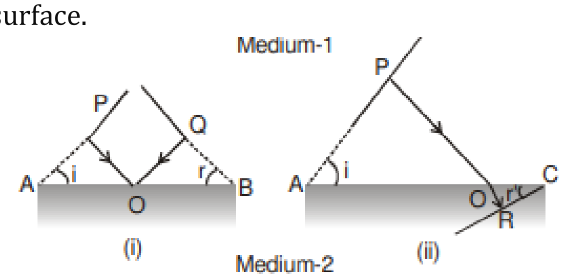
The preceding argument offers a compelling and intuitive way to comprehend how the refracted and reflected waves are constructed from secondary wavelets. Additionally, we can interpret the laws of reflection and refraction by considering that the time taken by light to travel along different rays from one wave front to another must remain consistent. Figure illustrates the incident and reflected wave fronts when a parallel beam of light encounters a flat surface. One specific ray, POQ, is depicted as perpendicular to both the reflected and incident wave fronts. The angle of incidence, denoted as 'i,' and the angle of reflection, denoted as 'r,' are defined as the angles formed by the incident and reflected rays concerning the normal. As illustrated in the figure, these angles are also equivalent to the angles between the wave front and the surface.



(a)



(b)



(Fig.)

(a) Huygens' construction for the Refracted wave.
(b) Calculation of propagation time between wave fronts in Reflected wave.

(i) Reflection
(ii) Refraction

Let's deduce the overall time taken to travel between wave fronts along the rays. Referring to Figure (c), we can express this as follows:

Total time for light to travel from P to Q

$$=\frac{PO}{V_1}+\frac{OQ}{V_1}=\frac{AO\sin i}{V_1}+\frac{OB\sin r}{V_1}=\frac{OA\sin i+(AB-OA)\sin r}{V_1}=\frac{AB\sin r+OA(\sin i-\sin r)}{V}$$

The different rays, each normal to the incident wave front, strike the surface at various points like O, resulting in different values of OA. In order to maintain uniformity of time, the right side of the equation needs to be independent of OA. This requires the coefficient of OA in Eq. (5) to be zero. In other words, sin i = sin r. Therefore, we have derived the law of reflection, i = r.

In the context of refraction at a plane surface that separates medium 1 (with the speed of light v_1) from medium 2 (with the speed of light v_2), we observe incident and refracted wavefronts making angles i and r' with the boundary. Angle r' is termed the angle of refraction. We also draw rays perpendicular to these wave fronts and proceed to calculate the time taken to travel between the wave fronts along any ray, as in the previous case.

Time taken from P to R $==\frac{PO}{V_1}+\frac{OR}{V_2}$

$$\frac{AO\sin i}{V_1}+\frac{(AC-OA)\sin r}{V_2}=\frac{AC\sin'}{V_2}+OA\left(\frac{\sin i}{V_1}-\frac{\sin r}{V_2}\right)$$

To maintain uniformity in the time it takes to travel between the wave fronts along different rays, the coefficient of OA in the equation should be zero, regardless of the specific ray being considered. Therefore, the equation for this case becomes:

$$\frac{\sin i}{\sin r_1}-\frac{V_1}{V_2}=n_{21}$$

Where n21 is a constant that depends on the refractive indices of the two media involved. This equation reflects the law of refraction and is commonly expressed as Snell's Law.

Snell's law of refraction, represented by the equation:

$$\frac{\sin i}{\sin r} = n_{21}$$

Where n_{21} is the refractive index of medium 2 concerning medium 1 . This equation defines the relationship between the angles of incidence and refraction when light transitions between two different media.

If the first medium is vacuum, we have an alternative form of Snell's law:

$$\frac{\sin i}{\sin r} = n_2$$

Where n_2 is the absolute refractive index of medium 2 concerning vacuum. A similar equation can be used to define the absolute refractive index, n1, for the first medium. From these equations, we can derive that n_{21} is equal to $\frac{V_1}{V_2}$, which is also equivalent to $\frac{n_2}{n_1}$, where V_1 and V_2 are the speeds of light in the respective media, and n_1 and n_2 are the refractive indices of the first and second media.

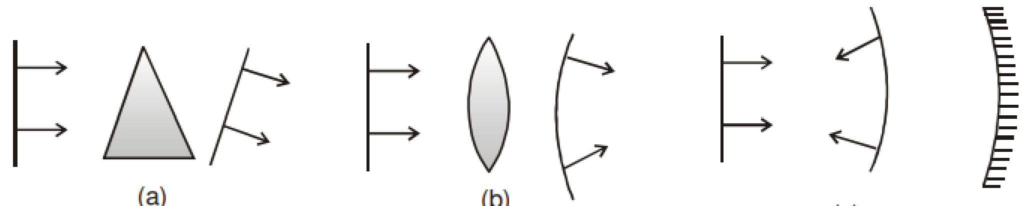
Once we have established the laws of reflection and refraction, we can gain a comprehensive understanding of the behavior of optical elements like prisms, lenses, and mirrors. These topics have been thoroughly discussed in the preceding chapter. In this section, we'll focus on explaining how wave fronts behave in the context of these three cases (as shown in the figures):

i. Let's consider a plane wave passing through a thin prism. It's quite evident that the part of the incoming wave front that traverses through the thickest portion of the glass gets delayed the most. This is due to the slower speed of light in glass, which explains the tilting of the emerging wave front.

ii. A concave mirror exhibits a similar phenomenon. The central region of the wave front has to travel a greater distance before and after reflection compared to the edge. This leads to the formation of a converging spherical wave front.

iii. A convex mirror also operates in a comparable manner. The central region of the wave front undergoes greater travel distances before and after reflection, resulting in the creation of a converging spherical wave front.

iv. Concave lenses and convex mirrors can be understood through time delay considerations in a similar manner. One interesting characteristic evident from the wave front diagrams is that the total time taken for light to travel from a point on the object to its corresponding point on the image remains consistent along any ray (as depicted in the figure). For instance, when a convex lens converges light to form a real image, it may appear that rays passing through the lens's center are shorter. However, due to the slower speed of light in glass, the time taken remains the same as for rays traveling near the lens's edge.



(a) (b) (c)