WAVE OPTICS

INTERFERENCE OF LIGHT WAVES AND YOUNG'S EXPERIMENT

INTERFERENCE:

Interference involves the combination of waves. When two or more waves overlap, they result in the sum of their individual displacements. Consider two waves originating from sources S1 and S2:

$y_1 = A_1 \sin(\omega t + kx_1)$
$y_2 = A_2 \sin(\omega t + kx_2)$

Here, A_1 and A_2 represent the amplitudes of the waves, ω is the angular frequency, and kx_1 and kx_2 denote their respective phase terms.

As a result of superposition, the combined displacement y_{net} can be expressed as the sum of the individual displacements:

$y_{\text{net}} = y_1 + y_2$
$y_{\text{net}} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$

Here, y_{net} represents the resultant displacement, A_1 and A_2 are the amplitudes of the waves, ω is the angular frequency, and kx_1 and kx_2 represent the respective phase terms.

The phase difference (Δφ) between y_1 and y_2 can be expressed as $\Delta\varphi = k(x_2 - x_1)$, where k represents a constant related to the wave, and $(x_2 - x_1)$ is the path difference.

This phase difference can also be related to the wavelength (λ) and the path difference (Δx) by the equation $\Delta\varphi = (2/\lambda)\,\Delta x$, where Δφ represents the phase difference, λ is the wavelength of the wave, and Δx is the path difference.

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \qquad (\text{As } I \propto A^2)$$

When the two displacements are in phase, then the resultant amplitude will be sum of the two amplitude & I_{net} will be maximum, this is known of constructive interference. For I_{net} to be maximum

$\cos\varphi = 1 \Rightarrow \varphi = 2n\pi$ where n = {0, 1, 2, 3, 4, 5..........}

$\frac{2\pi}{\lambda}\triangle x = 2n\pi \Rightarrow \Delta x = n\lambda$

For constructive interference

$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

When

$I_1 = I_2 = I$

$I_{net} = 4\ I$

$A_{net} = A_1 + A_2$

When two superimposed waves are in opposite phases, the resultant amplitude is obtained by taking the difference between the amplitudes of the two waves. In this situation, the combined wave, denoted as I_{net}, reaches its minimum value. This phenomenon is referred to as destructive interference.

For I_{net} to be minimum,

$\cos \Delta\varphi = -1$

$\Delta\varphi = (2n + 1)\ \pi$ where $n = \{0, 1, 2, 3, 4, 5..........\}$

$(2n + 1)\ \pi \Rightarrow \triangle x = (2n+1)\frac{\lambda}{2}$

For destructive interface

$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

If $I_1 = I_2$

$I_{net} = 0$

$A_{net} = A_1 - A_2$

Generally,

$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

If $I_1 = I_2 = I$

$I_{net} = 2I + 2I\cos\varphi$

$I_{net} = 2I\ (1 + \cos\varphi) = 4I\cos^2\frac{\triangle\phi}{2}$

Ratio of

$$I_{\max} \,\&\, I_{\min} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

Example.

Light originating from two sources, both of identical frequency and propagating in the same direction, exhibits interference, with intensities in a 4:1 ratio. Determine the ratio of the maximum intensity to the minimum intensity.

Solution.

$$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right) = \left(\frac{2+1}{2-1}\right) = 9:1$$

Bandwidth

The majority of signal transmission involves electromagnetic or radio waves. As we've discussed in topics like Amplitude Modulation and Frequency Modulation, the original signal is either superimposed on a carrier signal or alters its frequency to convey information from the sender to the receiver. At the receiving end, this information is demodulated and transformed back into the original signal.

Every signal comprises a multitude of wavelengths with various frequencies, making the signal unique in terms of its composition. This leads us to the techniques used for identifying signals. This is where the concept of "Bandwidth" becomes significant. So, what precisely is the Bandwidth of a signal?

Bandwidth of Signals

The bandwidth of a signal is the range between the highest and lowest frequencies it contains. In other words, it's the difference between the upper frequency (fH) and the lower frequency (fL) of the signal. Bandwidth is typically measured in Hertz (Hz), which is the unit for frequency.

To illustrate this concept, consider a common scenario when tuning a radio. When you tune a radio, you encounter various stations at specific frequencies. For instance, the bandwidth of FM radio typically ranges from 88.1 MHz to 101.1 MHz, covering a bandwidth of 200 KHz in most areas. Each radio station operates at its own unique frequency within this range, serving as a kind of identification.

The diagram above depicts various bandwidths of electromagnetic waves, and this information is crucial for identifying the type of wave based on its frequency and, subsequently, understanding its applications. Bandwidths are relevant for various types of waves, both in the visual and audio spectrums, and even beyond them.

For instance, the audible bandwidth of the human ear ranges from 20 Hz to 20,000 Hz. Sounds below 20 Hz are referred to as infrasonic, while sounds above 20,000 Hz are known as ultrasonic. Dogs can hear ultrasonic sounds, and blue whales can produce infrasonic sounds. The concept of bandwidth has numerous applications in different frequency ranges and domains.

Applications of Signal Bandwidth

Ultrasound

Ultrasound, a technique employing sound signals at frequencies exceeding 20,000Hz, finds valuable application in the field of medicine for assessing the well-being and status of internal organs, as well as for monitoring fetal growth during pregnancy. This non-invasive imaging method operates on the principle of high-frequency sound waves, which are emitted into the body and then bounce back to a transducer, creating detailed images of the internal structures. Medical professionals utilize ultrasound for a variety of diagnostic and monitoring purposes, such as inspecting organ health, assessing potential abnormalities, and tracking the developmental progress of a growing fetus in the womb.

Radar

The abbreviation RADAR stands for "Radio Detection and Ranging." RADAR technology employs extremely high-frequency sound waves, typically in the range of 1 to 3 MHz, and it finds application in a wide array of fields, including space exploration, defense, engineering, and materials analysis.

With this in mind, we have explored the concept of signal bandwidth and delved into its practical applications and associated constraints. A thorough grasp of the ideas presented in this discussion will assist the reader in comprehending the intricate workings and operational prerequisites of real-world systems.

Example.

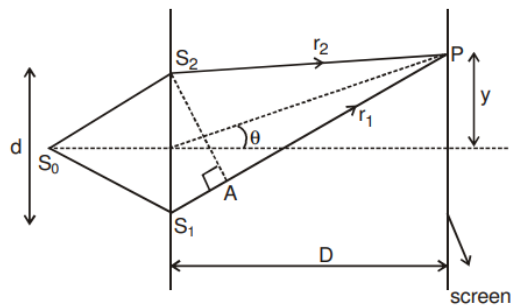
> Calculate the spectral bandwidth of the specified signal characterized by the frequencies 3125 MHz, 635 MHz, 2000 MHz, and 7000 MHz.

Solution.

> The bandwidth of a signal is defined as the discrepancy between its uppermost and lowermost frequencies. In the context of the provided system, the bandwidth computes to 6365 MHz, obtained by subtracting 635 MHz from 7000 MHz. Hence, it may be conclusively stated that the signal's bandwidth is 6365 MHz.

Analysis of Interference Pattern

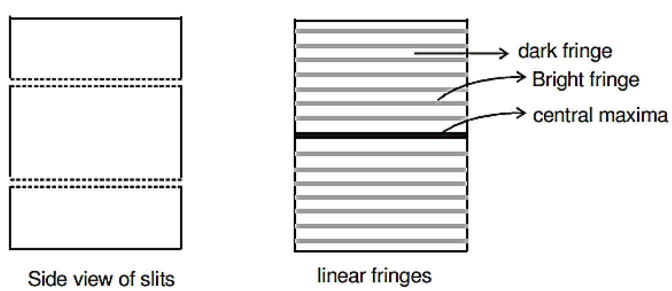
In the configuration described above, our foremost concern has been to ensure that the incident light waves traveling through S1 and S2 remain in phase with each other. However, it is crucial to acknowledge that when these waves reach point P, they may not be in perfect phase alignment. This discrepancy arises from the fact that the wave traveling from S1 to P follows a longer path compared to the wave emanating from S2 and reaching P. The phenomenon of phase difference due to this path disparity has been previously expounded upon. When the path difference is either zero or an integer multiple of the wavelength, the arriving waves synchronize perfectly and undergo constructive interference, resulting in heightened intensity at the observation point. Conversely, if the path difference equals an odd multiple of half a wavelength, the waves arrive out of phase and undergo complete destructive interference, leading to diminished intensity. Hence, it is the path difference denoted as Δx that unequivocally dictates the intensity observed at point P.



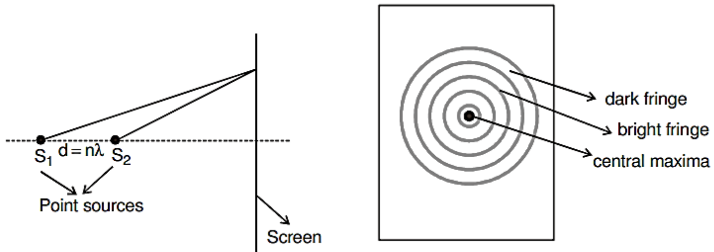
Path difference $\triangle p = s_1 p - s_2 p = \sqrt{\left(y+\frac{d^2}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$ ... (1)

SHAPE OF INTERFERANCE PATTERN

1. The Configuration of the Interference Pattern Arising from the Superposition of Waves Generated by Two Slits.

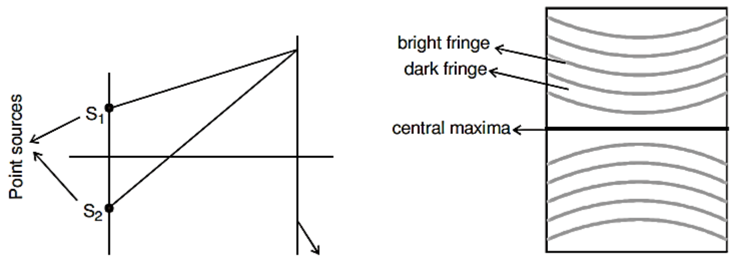


2. The Geometric Form of the Interference Pattern Resulting from the Interplay of Waves Emanating from Two Point Sources Arranged Orthogonally to the Screen.



Concentric Circular fringes

3. The Configuration of the Interference Pattern Arising from the Superposition of Waves Generated by Two Point Sources Positioned in a Line Parallel to the Screen.



Hyperbolic fringes