ELECTRIC CHARGE AND FIELD

GAUSS'S LAW

GAUSS'S LAW IN ELECTROSTATICS OR GAUSS'S THEOREM

This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions

Statement and Details:

Gauss's law is stated as given below. The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

If S is the Gaussian surface and $\sum_{i=1}^{n} q_i$ is the total charge enclosed by the Gaussian surface, then according to Gauss's law,

$$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

Note:

(i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on total charge present inside gaussian surface.

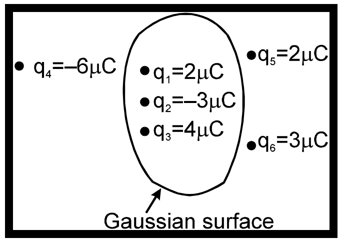
(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.

(iv) Electric field intensity at the gaussian surface is due to all the charges present inside as well as outside the gaussian surface.

(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a gaussian surface $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$

Example Find out flux through the given gaussian surface.



Solution. $S = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3 \times 10^{-6}}{\varepsilon_0}$ Nm²/C

Example If a point charge q is placed at the center of a cube, then find out flux through any one surface of cube.

Solution. Flux through 6 surfaces $= \frac{q}{\varepsilon_0}$. Since all the surfaces are symmetrical

so, flux through one surface $= \frac{1}{6}\frac{q}{\varepsilon_0}$

Finding electric field from Gauss`s Theorem:

From gauss`s theorem, we can say

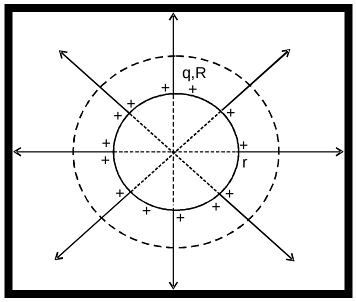
$$\int \vec{E}.\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0}$$

Finding E due to a spherical shell:
Electric field outside the Sphere:

Since, electric field due to a shell will be radially outwards.

So let's choose a spherical Gaussian surface Applying

Gauss`s theorem for this spherical Gauss`s surface,



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$ (because the $\vec{E}$ is normal to the surface)

$\downarrow$

$E\int ds$ (because value of E is constant at the surface)

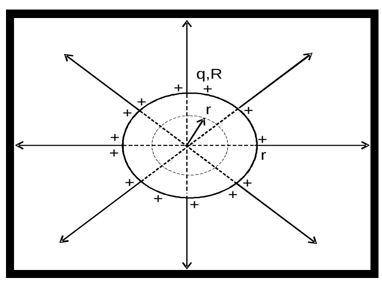
$E\,(4\pi r^2)$ ($\int ds$ total area of the spherical surface = $4\,\pi r^2$)

$\Rightarrow \quad E\,(4\pi r^2) = \dfrac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad E_{out} = \dfrac{q}{4\pi\varepsilon_o r^2}$

Electric field inside a spherical shell:

Let's choose a spherical gaussian surface inside the shell.

Applying Gauss`s theorem for this surface



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0} = 0$$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$

$\downarrow$

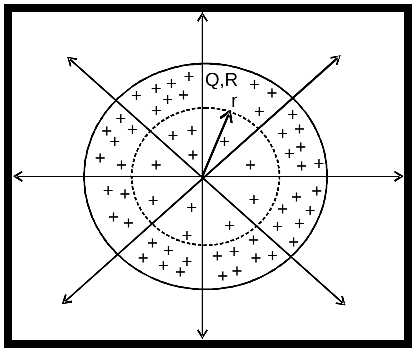
$E\int ds$

$\downarrow$

$E\,(4\pi r^2) \Rightarrow \quad E\,(4\pi r^2) = 0 \quad \Rightarrow \quad E_{in} = 0$

Electric field inside a solid sphere:

For this choose a spherical gaussian surface inside the solid sphere Applying gauss`s theorem for this surface.



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_o} = \frac{qr^3}{\varepsilon_o R^3}$$

$\downarrow$

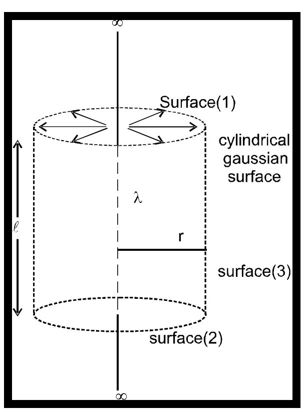
$\int E\, ds$

$\downarrow$

$E\,(4\pi r^2) \Rightarrow E(4\pi r^2) = \dfrac{qr^3}{\varepsilon_o R^3}$

$E = \dfrac{q\ r}{4\pi\varepsilon_o R^3} \Rightarrow E_{in} = \dfrac{kQ}{R^3} r$

Electric field due to infinite line charge (having uniformly distributed charged of charge density λ):



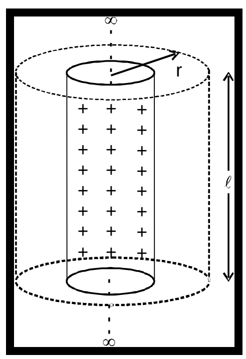
Electric field due to infinite wire is radial so we will choose cylindrical Gaussian surface as shown is figure.

$$\phi_{net} \rightarrow \phi_1 = 0, \quad \phi_2 = 0, \quad \phi_3 \neq 0 = \frac{q_{in}}{\varepsilon_o} = \frac{\lambda \ell}{\varepsilon_o}$$

$$\phi_3 = \int \vec{E}.\vec{ds} = \int E\, ds = E \int ds = E\,(2\pi r \lambda)$$

$$E\,(2\pi r \lambda) = \frac{\lambda \ell}{\varepsilon_o} \quad \Rightarrow \quad E = \frac{\lambda}{2\pi \varepsilon_o r} = \frac{2k\lambda}{r}$$

Electric field due to infinity long charged tube (having uniform surface charge densityσ and radius R)):



(i) **E out side the tube :-** lets choose a cylindrical gaussian surface

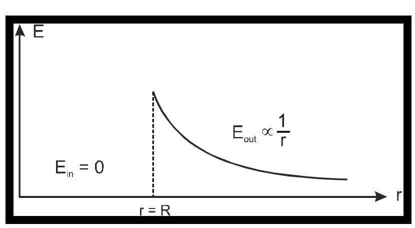
$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\sigma 2\pi R \ell}{\varepsilon_o} \quad \Rightarrow \quad E_{out} \times 2\pi r \lambda = \frac{\sigma 2\pi R \ell}{\varepsilon_o} \Rightarrow \quad E = \frac{\sigma R}{r\, \varepsilon_0}$$

(ii) **E inside the tube :**

lets choose a cylindrical gaussian surface in side the tube.

$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = 0$$

So $E_{in} = 0$



E due to infinitely long solid cylinder of radius R (having uniformly distributed charge in volume (charge density ρ)):

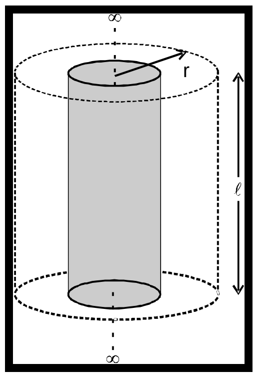
(i) E at outside point: -

Let's choose a cylindrical gaussian surface.

Applying gauss`s theorem

$$E \times 2\pi r\lambda = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 \ell}{\varepsilon_o}$$

$$E_{out} = \frac{\rho R^2}{2r\ \varepsilon_0}$$



(ii) E at inside point:

let's choose a cylindrical gaussian surface inside the solid cylinder.

Applying gauss`s theorem $E \times 2\pi r\lambda = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 \ell}{\varepsilon_o}$

$$E_{in} = \frac{\rho r}{2\varepsilon_0}$$