ELECTRIC CHARGE AND FIELD

FORCES BETWEEN MULTIPLE CHARGES

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The mutual electric force between two charges is given by Coulomb's law. How to calculate the force on a charge where there are not one but several charges around? Consider a system of n stationary charges q_1 , q_2 , q_3 , ..., q_n in vacuum. What is the force on q_1 due to q_2 , q_3 , ..., q_n ? Coulomb's law is not enough to answer this question. Recall that forces of mechanical origin add according to the parallelogram law of addition. Is the same true for forces of electrostatic origin?

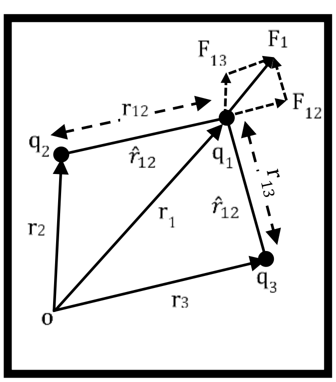
Experimentally, it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition.

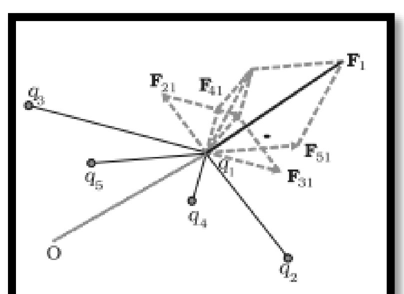
To better understand the concept, consider a system of three charges q_1, q_2 and q_3 , as. The force on one charge, say q_1 , due to two other charges q_2 , q_3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges. Thus, if the force on q_1 due to q_2 is denoted by F_{12}, F_{12} Even though other charges are present. Thus,

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} r_{12}$$

In the same way, the force on q1 due to q_3 , denoted by F_{13}, is given by

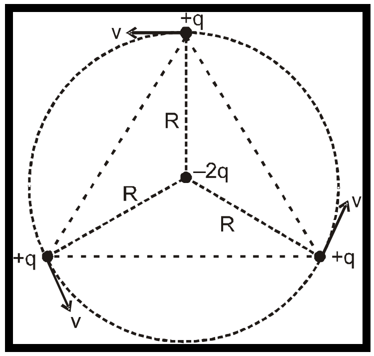
$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} r_{13}$$



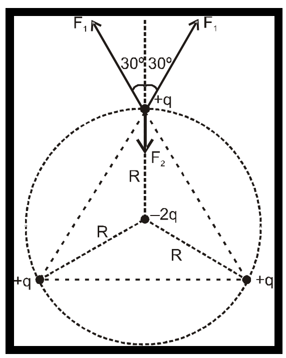


(b)

Example Three equal point charges of charge +q are moving along a circle of radius R and a point charge –2q is also placed at the center of circle as (figure), if charges are
revolving with constant and same speed then calculate speed



Solution.



$$F_2 - 2F_1 \cos 30 = \frac{mv^2}{R} \Rightarrow \frac{K(q)(2q)}{R^2} - \frac{2(Kq^2)}{(\sqrt{3}R)^2}\cos 30 = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{kq^2}{Rm}\left[2 - \frac{1}{\sqrt{3}}\right]}$$

Example Two equally charged identical small metallic spheres A and B repel each other with a force 2×10^{-5}N when placed in air (neglect gravitation attraction). Another identical uncharged sphere C is touched to B and then placed at the mid point of line joining A and B. What is the net electrostatic force on C?

Solution. Let initially the charge on each sphere be q and separation between their centres be r; then according to given problem.

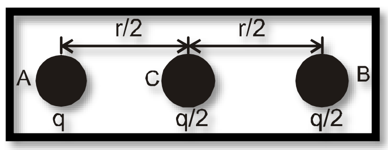
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{r^2} = 2 \times 10^{-5}\ \text{N}$$

When sphere C touches B, the charge of B, q will distribute equally on B and C as sphere are identical conductors, i.e., now charges on spheres;

$q_B = q_C = (q/2)$

So sphere C will experience a force

$$F_{CA} = \frac{1}{4\pi\varepsilon_0}\frac{q(q/2)}{(r/2)^2} = 2F \text{ along } \overrightarrow{AB} \text{ due to charge}$$



on A

and $$F_{CB} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(q/2)}{(r/2)^2} = F \text{ along } \overrightarrow{BA} \text{ due to charge on B}$$

So the net force F_C on C due to charges on A and B,

$$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5}\ \text{N along } \overrightarrow{AB}.$$

Example Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value – q coulomb placed at the centre of the hexagon?

Solution. **Method-I :**

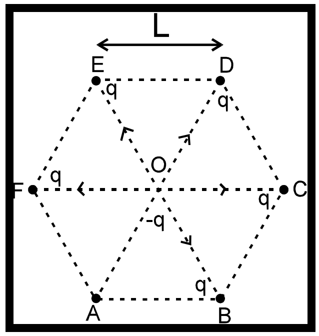
If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O would be zero (as the forces due to individual charges will balance each other), i.e. $\overrightarrow{F_R} = 0$

Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$\vec{F} + \vec{f} = 0$ i.e. $\vec{F} = -\vec{f}$

or $$|F| = |f| = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$$

$$\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2} \text{ along OD}$$



Method: II

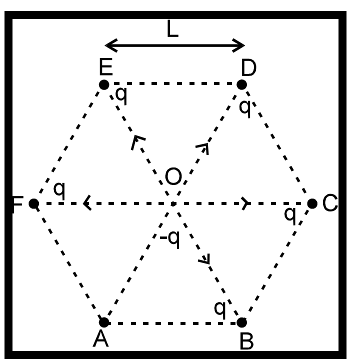
In the diagram we can see that force due to charge A and D are opposite to each other

$$\vec{F}_{DO} + \vec{F}_{AO} = 0 \quad \text{....(i)}$$

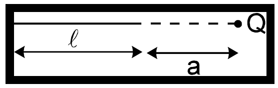
Similarly $$\vec{F}_{BO} + \vec{F}_{EO} = 0 \quad \text{....(ii)}$$

So $$\vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{CO} + \vec{F}_{DO} + \vec{F}_{EO} = \vec{F}_{Net}$$

Using (i) and (ii) $\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$ along OD



Example. A thin straight rod of length l carrying a uniformly distributed change q is located vacuum. Find the magnitude of the electric force on a point charge 'Q' kept as shown the figure.



Solution. As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part called element of length dy can be considered as point charge.

Charge on element $dq = \lambda dy = \frac{q}{\ell} dy$

Electric force on 'Q' due to element $= \frac{K.dq.Q}{y^2} = \frac{K.Q.q.dy}{y^2.\ell}$

All forces are along the same direction

$\therefore \quad F = \sum dF$ This sum can be calculated using integration,

therefore $F = \int_{y=a}^{a+\ell} \frac{KQqdy}{y^2\ell} = \frac{KqQ}{\ell}\left[-\frac{1}{y}\right]_a^{a+\ell} = \frac{KQ.q}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right] = \frac{KQq}{a(a+\ell)}$

Note: (1) The total charge of the rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.

Note: (2) If a >> l then $F = \frac{KQq}{a^2}$

behavior of the rod is just like a point charge.