ELECTRIC CHARGE AND FIELD

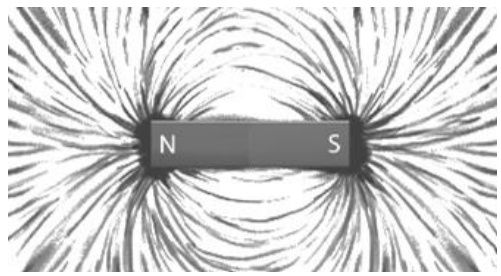
ELECTRIC FIELD LINES (FROM PRINCIPAL OF SUPER POSITIVE)

Electric field due to a system of charges (From principal of super positive):

Faraday's Lines of Force:

When any two point charges of equal magnitude, Q, are separated by a distance r at some region in space, they exert force on each other according to Coulomb's law. Now, we need to understand how charges exert force on each other without being in contact (this is also known as action at a distance). This is answered by Michael Faraday in his research paper named 'Experimental Researches in Electricity'. He did a simple experiment, which is as follows:

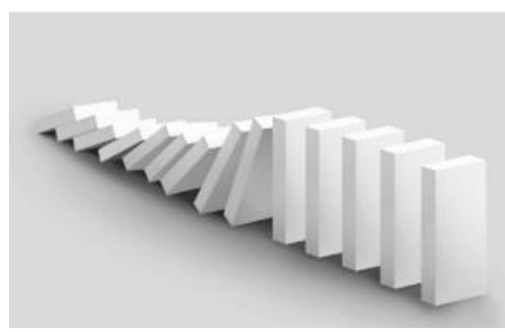
A bar magnet was placed under light on a dry paper and some iron filings were spread on the paper. It was seen that the iron filings were arranged in a particular way on the paper instead of in a random way as shown in the figure.



In this figure, the bar magnet (mentioned by NS) was placed under the paper. The curves that are seen around the magnet is the pattern that the iron filings got on the paper. Although the iron filings were not in contact with the bar magnet, they were influenced by something that the bar magnet produced.

From this experiment, Michael Faraday concluded the following points:

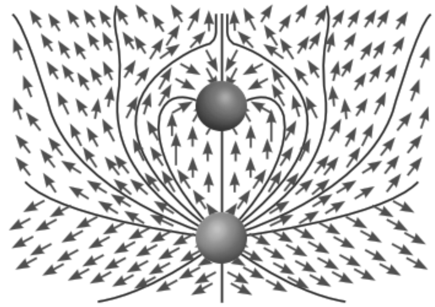
1. Faraday's belief was that force transmits through an action of continuous contact and this contact is provided by a field that is induced in space due to the presence of objects. The term 'action of continuous contact' can be understood by visualising an army of dominos between two charged particles. The force due to one charged particle gets transmitted through the dominos, and finally reaches the other charged particle.



2. Faraday termed the pattern obtained from the experiment with iron filings as lines of force, which were later known as field lines.

3. The patterns of the iron fillings around the magnet represent the **magnetic fields**.

When an experiment similar to the one with iron filings and a bar magnet was done with electric charges, a similar type of pattern was observed. This pattern is known as electric fields. The pattern is shown in the figure.



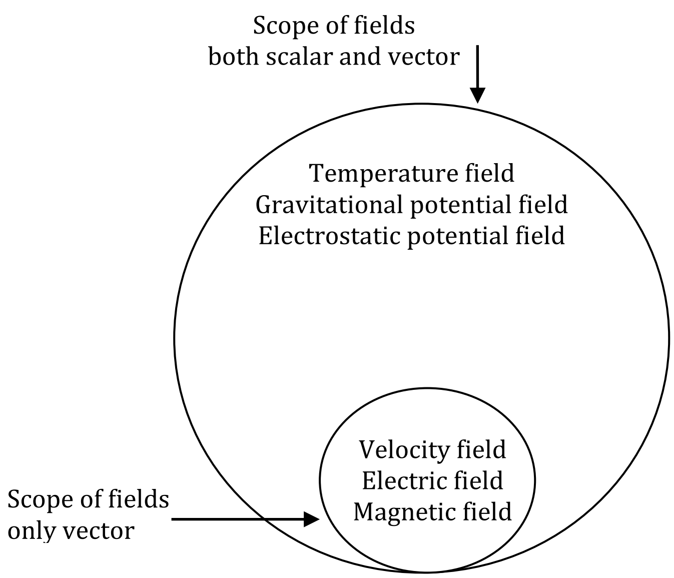
From the first conclusion, it becomes clear that the charges create their own field in the surroundings. Although there is no contact between the two charges in space, they exert force on each other through their fields whenever one charged particle comes in the field of another. This is known as action at a distance.

Fields and field lines:

Fields are like any other physical quantities, describing both scalars and vectors.

Qualitatively, in order to explain any non-contact forces between two objects or in order to describe the 'action at a distance' phenomenon, we have to imagine a field, irrespective of the nature of the non-contact forces.

The field may be vector or scalar, and it should be remembered that field lines represent vector fields (only vector quantities) as they denote the direction of action or force.



Electric Field:

Electric field is a region around a charged particle or object within which a force would be exerted on other charged particles or objects.

The force that is exerted by the source charge on the test charge is a two-step process:

1. At first, the source charge creates its own field, which means that it creates a region up to which it will be able to exert force on any other charged particle or object.

2. Whenever the test charge comes in that region, it feels the force due to the source charge.

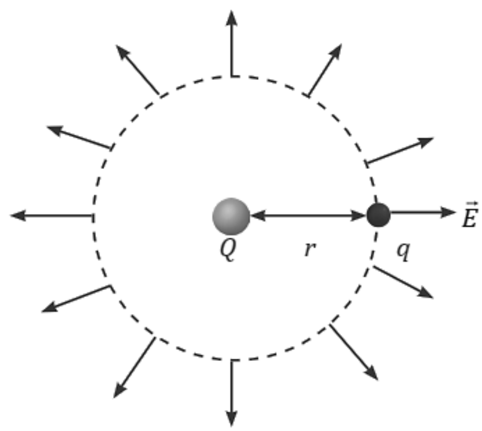
The term 'field' signifies how some distributed quantity (scalar or vector) varies with position

Electric field strength$(\vec{E})$:

The electric field strength (often simply called electric field) at a point is defined as the electrostatic force F_e per unit positive charge at that point.

Electric field strength due to a point charge at a distance:

Consider a source charge Q (it creates the field) and a test charge q (it feels the force) as shown in the figure. Let the separation between them be r.



The magnitude of electrostatic force on the test charge q is, $F_e = \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2}$. The magnitude of this electrostatic force will remain unchanged as long as the test charge is at the surface of a sphere of radius r. The field from the source charge can be visualised as light spreading out from a bulb in all directions.

The electric field strength or the electric field of a charge Q at a distance r is given by,

$$E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

SI unit: The SI unit of the electric field is NC^{-1}.

1. The nature of the electric field produced by a point charge is non-uniform because at every point in space, even though the magnitude is the same, the direction of the electric field is different.

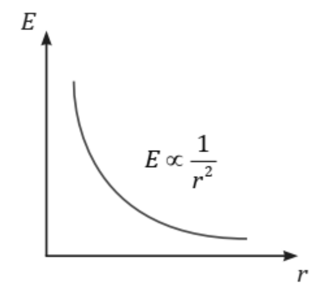
2. The electric field of a charge Q at a distance r is given by, $E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$

3. The direction of the electric field is radially outwards for a positive charge and radially inwards for a negative charge.

Graphical plot of electric field strength variation with distance from a point charge:

The electric field of a point charge Q at a distance r is given by,

$$E = \frac{F_e}{q} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$



Hence, it can be said that the electric field is inversely proportional to the square of the distance from the charge itself. Therefore, the E-r graph will be parabolic in nature as shown in the figure.

Ex. Calculate the electric field strength at a point 1 cm away from a point charge of magnitude, 10 μC. (Assume no other electric charge to be present)

Sol.

Given,
The charge is, Q = 10 μC.
The distance from the charge where the electric field strength is to be measured is, r = 1 cm = 0.01 m.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E = (9 \times 10^9)\left[\frac{10\times10^{-6}}{(0.01)^2}\right]$$

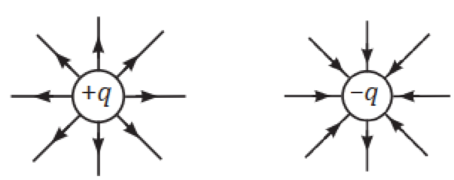
$$E = (9 \times 10^9)\left[\frac{10\times10^{-6}}{10^{-4}}\right]$$

$$E = 9 \times 10^8 NC^{-1}$$

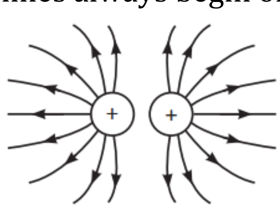
The direction of the electric field strength will be radially outward.

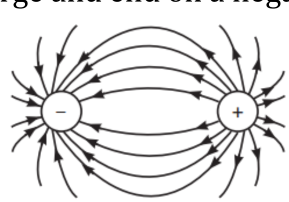
Properties of Electric Field Lines:

1. For a positive charge, the field lines will be radially outwards, and for a negative charge, the field lines will be radially inwards.

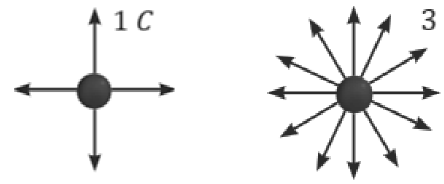


2. Electric field lines always begin on a positive charge and end on a negative charge.



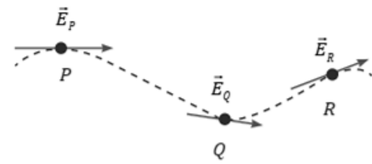


3. The number of lines leaving a positive charge or ending at a negative charge is proportional to the magnitude of the charge. The greater the magnitude of the charge, the more dense the field lines will be at the location of the charge.

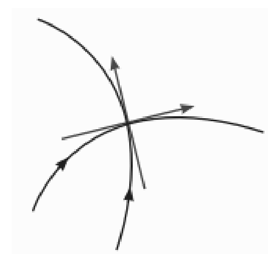


It should be noted that if 1 C charge and 3 C charge are considered, and 4 lines are used to represent the field lines for the 1 C charge, then 12 lines should be used to represent the field lines for the 3 C charge.

4. The tangent to a field line at any point gives us the direction of electric field at that point.

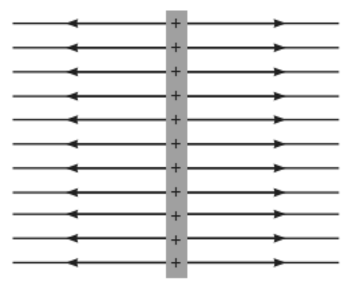


5. Two electric field lines can never intersect because if it happens, then there will be two different directions for a single value of electric field at the point of intersection of those two field lines, which is impossible.



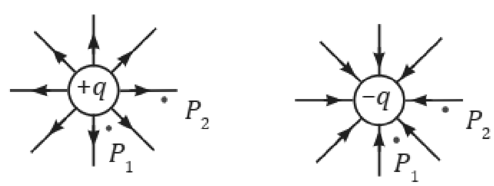
6. The field lines never form closed loops, as a line can never start and end on the same charge.

7. In a region of a uniform electric field, the field lines are straight, parallel, and uniformly spaced.



8. Let us take a case of a region of non-uniform electric field.

	AT P_1	AT P_2
Electric Field strength	High	Low

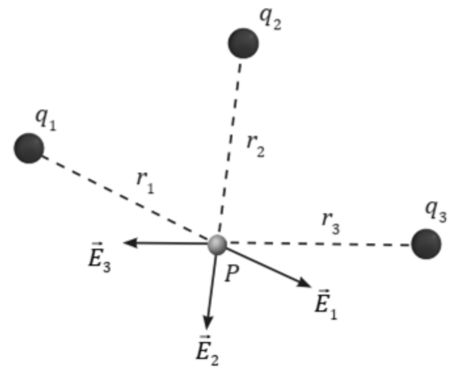


The strength of the electric field is greater where the density of the field lines is larger.

Principle of Superposition:

The resultant electric field at a point will be the vector sum of the electric fields due to all individual point charges.

Consider three point charges q_1, q_2, and q_3, as shown in the figure. If $\vec{E}_1, \vec{E}_2, \vec{E}_3$ are the electric fields due to q_1, q_2, and q_3 respectively, then the net electric field at point P is,

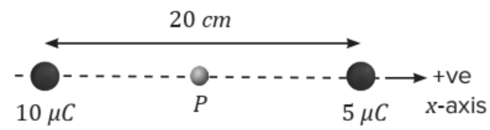


$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Therefore, if n number of charges are present in the space, then the electric field at point P will be,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots. + \vec{E}_n = \sum_{i=0}^{n} \vec{E}_i$$

Ex. Find the electric field at the midpoint of the line joining two charges separated by 20 cm.



Sol.

Let $q_1 = 10\mu C = 10 \times 10^{-6}$C and $q_2 = 5\mu C = 5 \times 10^{-6} C$

The distance of point P from both the charges is, r = 10 cm = 0.1 m. Therefore, at point P,

The electric field due to q_1 is, $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{i}$.

The electric field due to q_2 is, $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}(-\hat{i})$

Hence, the net electric field at point P is,

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_0}[\frac{q_1}{r^2} - \frac{q_2}{r^2}]\hat{i}$$

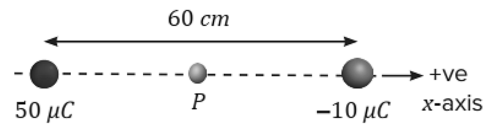
$$\vec{E}_{\text{net}} = (9 \times 10^9)[\frac{10\times10^{-6}}{(0.1)^2} - \frac{5\times10^{-6}}{(0.1)^2}]\hat{i}$$

$$\vec{E}_{net} = (9 \times 10^9)[10 - 5] \times \frac{10^{-6}}{(0.1)^2}\hat{i}$$

$$\vec{E}_{\text{net}} = (45 \times 10^9) \times \frac{10^{-6}}{10^{-2}}\hat{i}$$

$$\vec{E}_{net} = 45 \times 10^5 \hat{i} NC^{-1}$$

Ex. Find the electric field at the midpoint of the line joining two charges separated by 60 cm.



Sol.

Let $q_1 = 50\mu C = 50 \times 10^{-6} C$ and $q_2 = -10\mu C = -10 \times 10^{-6}$C

The distance of point P from both the charges is, r = 30 cm = 0.3 m.

Therefore, at point P,

The electric field due to q_1 is, $\vec{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{i}$

The electric field due to q_2 is, $\vec{E_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\hat{i}$

In this case, the directions of the electric field at point P due to both the charges are along the positive x-axis. Hence, the net electric field at point P is,

$$\vec{E}_{\text{net}} = \vec{E_1} + \vec{E_2}$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_0}[\frac{q_1}{r^2} + \frac{q_2}{r^2}]\hat{i}$$

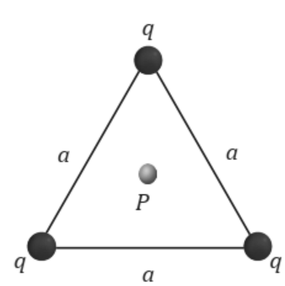
$$\vec{E}_{\text{net}} = (9 \times 10^9)[\frac{50\times10^{-6}}{(0.3)^2} + \frac{10\times10^{-6}}{(0.3)^2}]\hat{i}$$

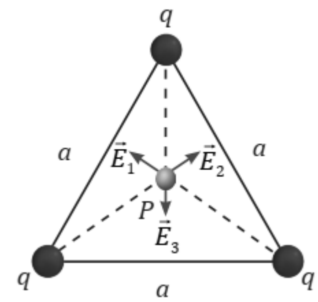
$$\vec{E}_{\text{net}} = (9 \times 10^9)[50 + 10] \times \frac{10^{-6}}{(0.3)^2}\hat{i}$$

$$\vec{E}_{net} = (9 \times 60 \times 10^9) \times \frac{10^{-6}}{9\times10^{-2}}\hat{i}$$

$$\vec{E}_{\text{net}} = 6 \times 10^6 \hat{i} NC^{-1}$$

Ex. Find the net electric field at P (at centroid)





Sol. The given triangle is an equilateral triangle of side length a, and we know that the centroid divides the altitude in the ratio of 2: 1. Hence, the distance of the centroid from any vertex of the triangle is $\frac{a}{\sqrt{3}}$.

The magnitude of $\vec{E}_1, \vec{E}_2, \vec{E}_3$, will be,

$$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(\frac{a}{\sqrt{3}})^2}$$

By symmetry, the net electric field at point P will be zero.

Short trick:

By polygon law, if we have any number of vectors forming the polygon, and they obey cyclic symmetry, the resultant of that vectors will always be zero.

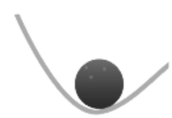
In this case, $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ can also form a triangle and they have a cyclic symmetry since the angle between each of them is 120°.

Equilibrium:

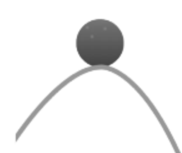
Equilibrium is a state for a mass/charge/particle when its state of motion remains unaffected, which implies that the net force acting on it is zero. Since the force is interconnected with the field, the previous statement also implies that the net field acting has to be zero.

Types of equilibrium:

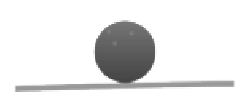
1. **Stable equilibrium:** In this case, if a particle is subjected to an impactive force, it tries to go back to its initial state of motion.



2. **Unstable equilibrium:** In this case, if a particle is subjected to an impactive force, it cannot go back to its initial state of motion.



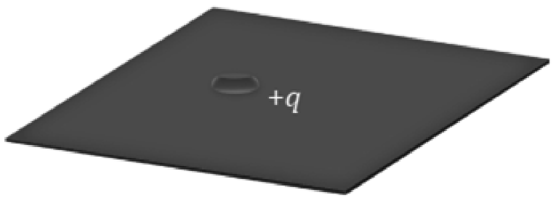
3. **Neutral equilibrium:** In this case, if a particle is subjected to an impactive force, it gets displaced, but its current and initial states of motion have no difference.

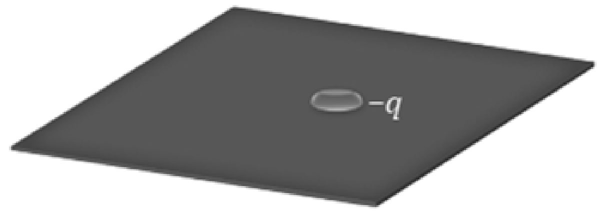


Layman's View of Geography of Electric Charges:

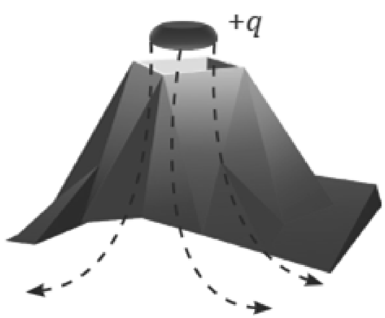
The electric charges also possess different states of equilibrium. To begin the discussion, we should remember that the electric field lines always begin on a positive charge and end on a negative charge.

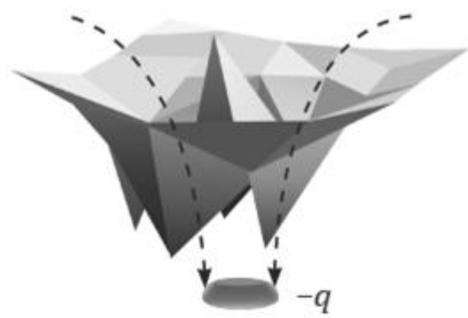
When we talk about positive and negative charges, we can consider them on the same level, as shown in the figure.



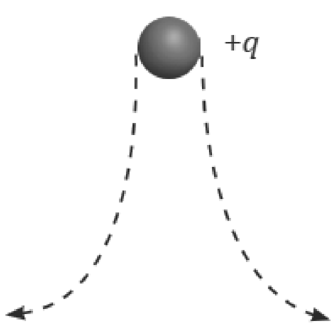


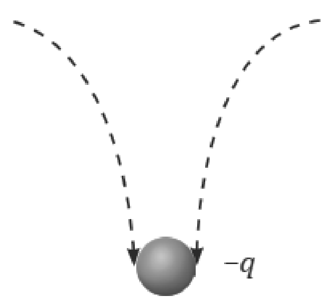
However, if the concept of field lines is incorporated with positive and negative charges, then the position of the positive charge can be thought of as the top of a hill from which a fountain originates. It is because field lines originate from the positive charge. The position of the negative charge can be thought of as a valley or the bottom of a funnel, because field lines originate from the positive charge and end at the negative charge.





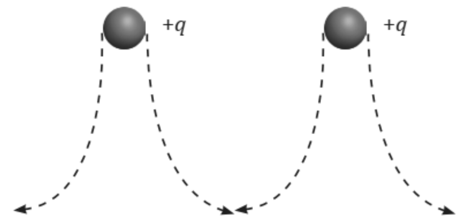
Hence, it can be said that the positive charge acts as the source and the negative charge acts as the sink. Therefore, it can be concluded that from the topological point of view, the positive charge and the negative charge are not at the same level. The side views of the geography of the charges are shown below.



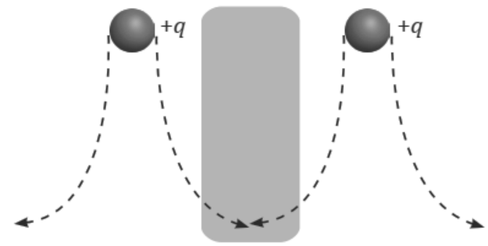


Equilibrium of electric charges:

• When two positive charges of an equal magnitude are placed side by side, from layman's view of the geography of charges, they can be seen as shown in the figure.

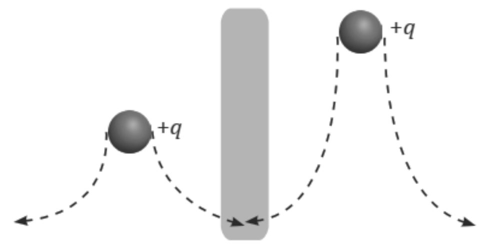


From the figure, the position of two positive charges of equal magnitude can be thought of as two different peaks of a hill of the same height. Now, the question arises that is there any possibility of equilibrium in this arrangement? Yes, the valley or region between the two peaks is the stable equilibrium position.

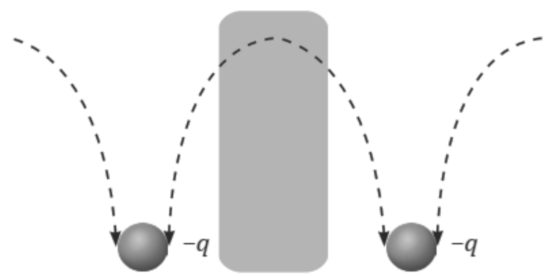


Zone of possibility for **stable equilibrium**

• Suppose that two positive charges of unequal magnitude are placed side by side. For this case as well, the valley or region between the two peaks is at the stable equilibrium position, but the region becomes smaller here and it is closer to the charge with less magnitude.



• When two negative charges of equal magnitude are placed side by side, from layman's view of the geography of charges, the position of the two negative charges of equal magnitude can be thought of as bases of funnels of the same depth. Now, the question arises that is there any possibility of equilibrium in this arrangement? Yes, the region (the peak) between the two charges is the unstable equilibrium position.

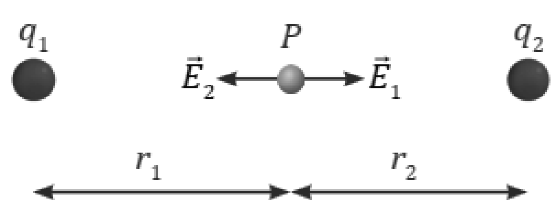


Zone of possibility for **unstable equilibrium**

Null Point:

The null point is a position where the net field turns out to be zero as a vector sum.

Consider two point charges q_1 and q_2 as shown in the figure. For the net field to be zero at point P, the following should be true:



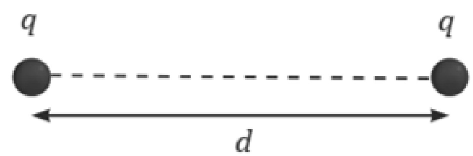
$$\vec{E}_{\text{net}} = \vec{0}$$

$$\Rightarrow \vec{E}_1 + \vec{E}_2 = \vec{0}$$

$\Rightarrow \frac{1}{4\pi\varepsilon_0}[\frac{q_1}{r_1^2} - \frac{q_2}{r_2^2}] = 0 \left[\begin{array}{c} The\ negative\ sign\ arises\ because\ the\ directions \\ of\ \vec{E}_1\ and\ \vec{E}_2\ are\ opposite. \end{array} \right]$

$\Rightarrow \frac{q_1}{r_1^2} = \frac{q_2}{r_2^2}$

Ex. Find the position along the line joining two point charges where the net electric field is zero.

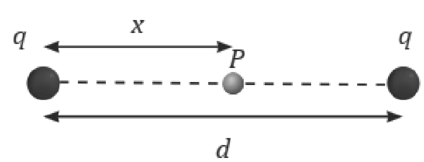


Sol. Let the electric field be zero at point P, which is x distance away from the charge on the left as shown in the figure.

Since both the charges are positive, the direction of the electric field on P due to these charges will be opposite.

Also, we assumed the net electric field at point P to be zero.
Thus,

$\vec{E}_{ngt} = \vec{0}$



$\Rightarrow |\vec{E}_1| = |\vec{E}_2|$

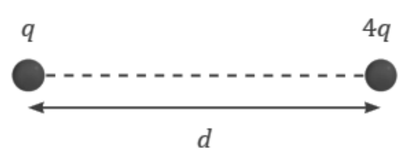
$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(d-x)^2}$

$\Rightarrow x = \pm(d - x)$

$\Rightarrow x = \frac{d}{2}$ [By taking only the positive value]

Therefore, the equilibrium position or the null point will be at the middle of the line joining the two equal positive charges.

Ex. Find the position along the line joining two point charges where the net electric field is zero.



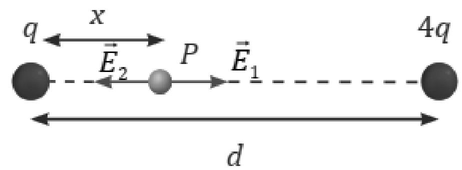
Sol. Let the electric field be zero at point P, which is x distance away from the charge at the left.

Since both the charges are positive, the direction of the electric field at P due to these charges will be opposite.

Also, we assumed the net electric field at point P to be zero.
Thus,

$\vec{E}_{\text{net}} = \vec{0}$

$\Rightarrow |\vec{E}_1| = |\vec{E}_2|$



$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{4q}{(d-x)^2}$

$\Rightarrow 4x^2 = (d-x)^2$

$\Rightarrow 2x = \pm(d-x)$

$\Rightarrow 2x = (d-x)$ [By taking only the positive value]

$\Rightarrow x = \frac{d}{3}$

Therefore, the equilibrium position or the null point will be closer to the smaller positive charge along the line joining the two unequal positive charges.