ELECTRIC CHARGE AND FIELD

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If two-point charges equal in magnitude q and opposite in sign separated by a distance a such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude $p = (q \times a)$ and direction from negative charge to positive charge.

Note:

[In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.] The C.G.S unit of electric dipole moment is **debye** which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} franklin and separation of 1 Å, i.e.,

1 debye (D) $= 10^{-10} \times 10^{-8} = 10^{-18}$ Fr × cm

$$1\text{ D} = 10^{-18} \times \frac{C}{3 \times 10^9} \times 10^{-2}\text{ m} = 3.3 \times 10^{-30}\text{ C} \times \text{m}.$$

S.I. Unit is coulomb × metre = C . m

Example A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, – 0.15 m) and B; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system?

Solution. Net charge $= 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment,

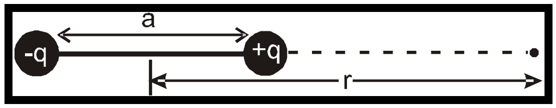
P = (Magnitude of charge) × (Separation between charges)

$= 2.5 \times 10^{-7}\,[0.15 + 0.15]$ C m $= 7.5 \times 10^{-8}$ C m

The direction of dipole moment is from B to A.

Electric Field Intensity Due to Dipole :

(i) At the axial point :



$$\vec{E} = \frac{Kq}{\left(r - \frac{a}{2}\right)} - \frac{Kq}{\left(r + \frac{a}{2}\right)^2} \text{ along the } \hat{P} = \frac{Kq(2ra)}{\left(r^2 - \frac{a^2}{4}\right)^2}\hat{P}$$

If $r >> a$ then $\vec{E} = \frac{Kq2ra}{r^4}\hat{P} = \frac{2K\vec{P}}{r^3}$,

As the direction of electric field at axial position is along the dipole moment ($\vec{P}$)

so $\vec{E}_{axial} = \frac{2K\vec{P}}{r^3}$

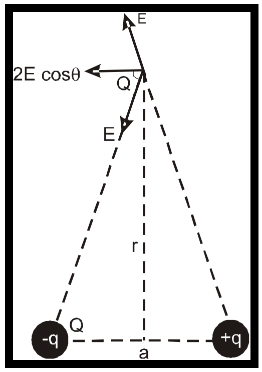
(ii) Electric field at perpendicular Bisector (Equitorial Position)

$E_{net} = 2\,E\cos\theta$ (along $-\hat{P}$)

$$\vec{E}_{net} = 2\left(\frac{Kq}{\left(\sqrt{r^2 + \left(\frac{a}{2}\right)^2}\right)^2}\right)\frac{\frac{a}{2}}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}}(-\hat{P}) = 2\frac{Kqa}{\left(r^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}}(-\hat{P})$$

If r >> a then

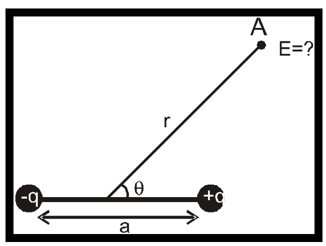
$$\vec{E}_{net} = \frac{KP}{r^3}(-\hat{P})$$



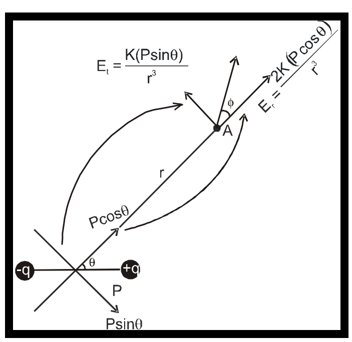
As the direction of $\vec{E}$ at equitorial position is opposite of $\vec{P}$ so we can write in vector form:

$$\vec{E}_{eqt} = -\frac{K\vec{P}}{r^3}$$

(iii) Electric field at general point (r, θ) :



For this, Lets resolve the dipole moment into components



One component is along radial line (=P cosθ) and other component is $\perp_r$ to the radial line (=Psinθ)

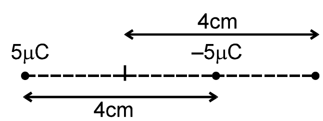
From the given figure

$$E_{net} = \sqrt{E_r^2 + E_t^2} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{E_t}{E_r} = \frac{\frac{KP\sin\theta}{r^3}}{\frac{2KP\cos\theta}{r^3}} = \frac{\tan\theta}{2}$$

$$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} \quad ; \qquad \tan\phi = \frac{\tan\theta}{2}$$

Example Two charges, each of 5 C but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.



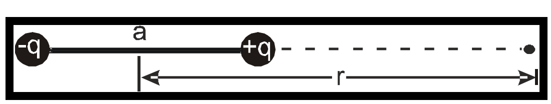
Solution. We can not use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

$q = 5 \times 10^{-6}$ C, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m

$$E_{res} = E_{+} + E_{-} = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6cm)^2} = \frac{144}{144\times 10^{-8}} \text{ NC}^{-1} = 10^8 \text{ N C}^{-1}$$

Electric Potential due to a small dipole :

(i) Potential at axial position :



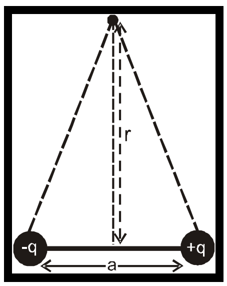
$$V = \frac{Kq}{\left(r - \frac{a}{2}\right)} + \frac{K(-q)}{\left(r + \frac{a}{2}\right)}$$

$$V = \frac{Kqa}{\left(r^2 - \left(\frac{a}{2}\right)^2\right)}$$ If r >> a than

$$V = \frac{Kqa}{r^2}$$ where qa = p $\Rightarrow$ $V_{axial} = \frac{KP}{r^2}$

(ii) Potential at equitorial position :

$$V = \frac{Kq}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} + \frac{K(-q)}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} = 0 \quad \Rightarrow \quad V_{eqt} = 0$$

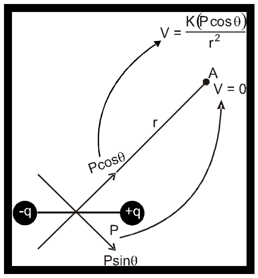


(iii) Potential at general point (r,θ) :

Lets resolve the dipole moment $\vec{P}$ into components Pcosθ component along radial line and Psinθ component $\perp_r$ to the radial line.

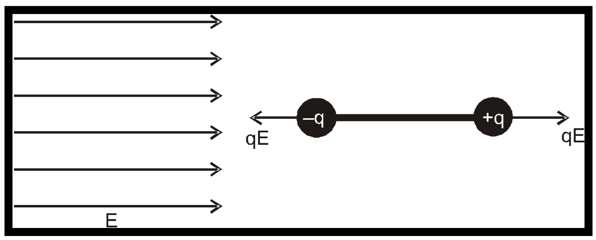
For the Pcosθ component, the point A is an axial point, so, potential at A due to Pcos component = $\frac{K(P\cos\theta)}{r^2}$ and for Psinθ component, the point A is an equatorial point, so potential at A due to Psinθ component = 0

$$V_{net} = \frac{K(P\cos\theta)}{r^2} \quad \Rightarrow \quad V = \frac{K\left(\vec{P}.\vec{r}\right)}{r^3}$$



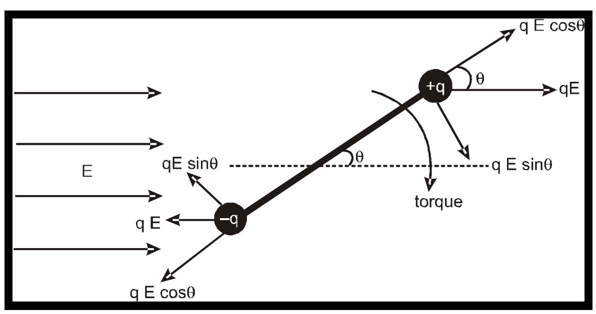
Dipole in uniform electric field

(i) Dipole is placed along electric field :



In this case $F_{net} = 0$, $\tau_{net} = 0$ so it is an equilibrium state. And it is a stable equilibrium position.

(ii) If the dipole is placed at θ angle from $\vec{E}$: -



In this case $F_{net} = 0$ but

Net torque $\tau = (qE\sin\theta)\ (a)$

Here $qa = P \quad \Rightarrow \quad \tau = PE\sin\theta$

in vector form $\vec{\tau} = \vec{P} \times \vec{E}$

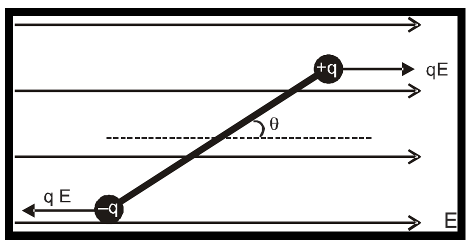
Example A dipole is formed by two point charge –q and +q, each of mass m, and both the point charges are connected by a rod of length λ and mass m_1. This dipole is placed in uniform electric field $\vec{E}$. If the dipole is disturbed by a small angle θ from stable equilibrium position, prove that its motion will be almost SHM. Also find its time period.

Solution. If the dipole is disturbed by θ angle,

$\tau_{net} = -PE\sin\theta$ (here – ve sign indicates that direction of torque is opposite of θ)

If θ is very small, $\sin\theta = \theta \Rightarrow \tau_{net} = -(PE)\theta$

$\tau_{net} \propto (-\theta)$ so motion will be almost SHM. $T = 2\pi\sqrt{\frac{I}{K}}$



(iii) Potential energy of a dipole placed in uniform electric field :

$U_B - U_A = -\int_A^B \vec{F}\cdot\vec{dr}$ Here $U_B - U_A = -\int_A^B \vec{\tau}\cdot\vec{d\theta}$

In the case of dipole, at $\theta = 90°$, P.E. is assumed to be zero.

$$U_\theta - U_{90°} = -\int_{\theta=90°}^{\theta=\theta} (-PE\sin\theta)(d\theta)$$

(As the direction of torque is opposite of θ)

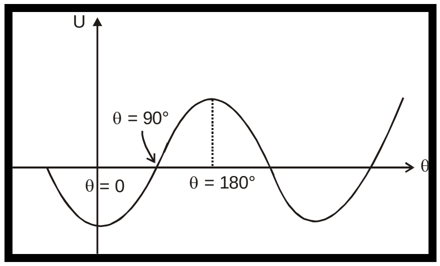
$U_\theta - 0 = -PE\cos\theta \Rightarrow$ $\theta = 90°$ is chosen as reference, so that the lower limit comes out to be zero.

$U_\theta = -\vec{P}\cdot\vec{E}$

From the potential energy curve, we can conclude :

(i) at $\theta = 0$, there is minimum of P.E. so it is a stable equilibrium position.

(ii) at $\theta = 180°$, there is maxima of P.E. so it is a position of unstable equilibrium.



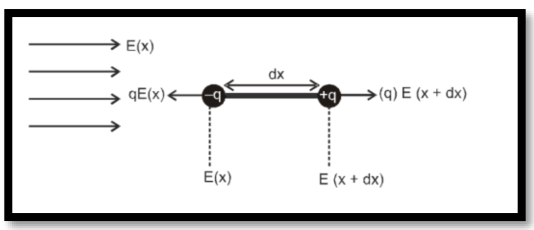
Dipole in nonuniform electric field :

(If the dipole is placed in the along $\vec{E}$)

Net force on the dipole $F_{net} = q\,E(x + dx) - q\,E(x)$

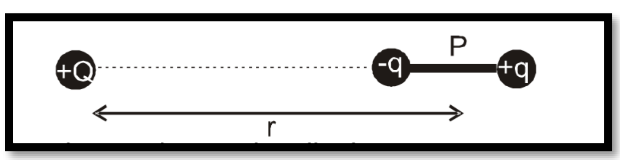
$F_{net} = q\,\dfrac{E(x+dx)-E(x)}{dx}$ (dx) here q (dx) = P

$F_{net} = P\left(\dfrac{dE}{dx}\right)$



Force between a dipole and a point charge :

Example A short dipole of dipole moment P is placed near a point charge as shown in figure. Find force on the dipole due to the point charge



Solution. Force on the point charge due to the dipole

$F = (Q)\,E_{dipole}$

$F = (Q)\left(\dfrac{2KP}{r^3}\right)$ (right)

From action reaction concept, force on the dipole due to point charge will also be

$F = \frac{2KPQ}{r^3}$ (left)