Electric charge and field

Coulomb's Law

COULOMB'S LAW:

Charles-Augustin de coulomb discovered this law, which states that the force between **two Static point charges** is inversely proportional to the square of the distance between the Charges and is directly proportional to the product of the magnitude of the two charges and the force acts along the line joining the two charges.

Mathematically, if Q_1 and Q_2 are two point charges separated by distance r, then the Electrostatic force between them is,

$$F \propto Q_1 Q_2 \text{ ………. (i)}$$

$$F \propto \frac{1}{r^2} \text{ ………. (ii)}$$

By combining the equation and equation, we get,

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{k_e Q_1 Q_2}{r^2}$$

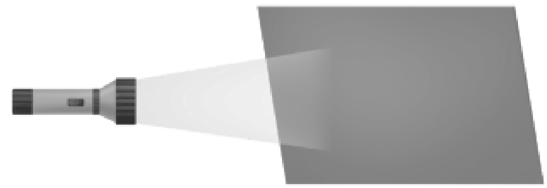
Where k_e is the proportionality constant known as coulomb's constant the electrostatic force (f) as defined looks similar to the gravitational force between two masses defined by $F = \frac{GM_1M_2}{r^2}$. The comparison between the expression of the electrostatic force and the gravitational force is given.

Quantity	Electrostatic force	Gravitational force
Expression of force	$F = \frac{GM_1M_2}{r^2}$	$F = \frac{k_e Q_1 Q_2}{r^2}$
Applicability	Force of interaction between static point charges	Force of interaction between masses
Nature	Attractive or repulsive	Always attractive
Comparable strength	Stronger	Weaker
Nature of proportionality constant	It can be shielded that means depends on the medium	Universal

Coulomb's constant:

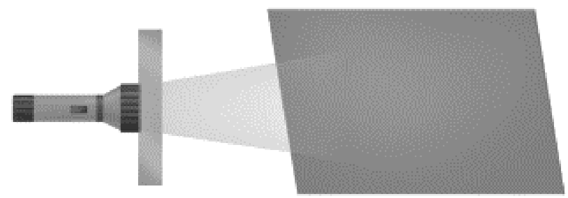
It does not matter where the masses are; be it at the ground, underwater, or even outer space, the gravitational force is always the same, but it is not true for the electrostatic force. It is different in different mediums.

Consider that a light beam from a torch is falling on a surface as shown in the figure.



Now, if the medium is changed, i.e., we put something in between the torch and the surface (say a glass slab), the intensity of light falling on the surface gets reduced.

Similarly, the electrostatic force also varies with the medium, and the way it varies is given by the constant ke we saw earlier. From this discussion it may seem to you that it is not a constant after all. Then why is it known as a constant? It is because it is constant for a given medium.



The coulomb constant k_e is defined as $k_e = \frac{1}{4\pi\varepsilon}$ where ε is known as the permittivity of the medium, and it is the deciding factor of the coulomb's constant.

In the case of vacuum, the permittivity of the medium is given by,

$$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$$

therefore, the value of coulomb constant at vacuum is,

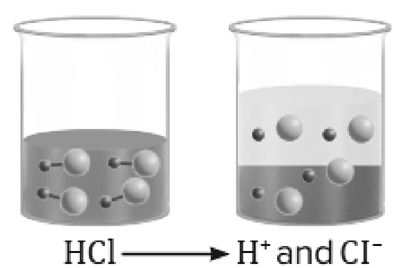
$$k_e = 9 \times 10^9 Nm^2C^{-2}$$

Relative Permittivity:

If the permittivity of vacuum is denoted by ε_0 and the permittivity of any medium is denoted by ε, then the relative permittivity (or dielectric constant) of the given medium is defined by the following expression:

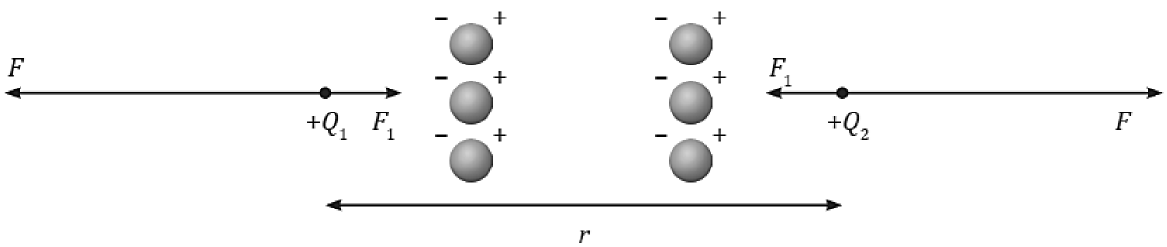
$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

The physical significance of the relative permittivity or the dielectric constant is the ability of a medium to ionise whatever is put inside it. For example, when hydrochloric acid (HCL) is mixed with water, the hcl molecule gets ionised as H^+ and Cl^- because the dielectric constant for water is 81. Hence, the electrostatic force between the ions is reduced by 81.



Force experienced by two point charges placed in a medium:

It may seem that the force between two point charges placed in a medium other than the vacuum changes from what was the force between them in vacuum. However, this statement is incomplete. The correct statement is that **the force between two charges placed in a medium other than vacuum does not change from what was the force between them in vacuum, but the force experienced by each charge alone in totality changes**. To understand this statement clearly, let us consider the scenario shown in the figure.



In This Figure, The Yellow Particles Are The Charged Particles Of The Medium. Here, F is The Force on Q_1 Due To Q_2 and Vice Versa, And F_1 Is the Force on the Charges by the Medium.
Therefore, the net force on the charge is,

$$F_{net} = (F - F_1) < \frac{k_e Q_1 Q_2}{r^2}$$

Vector form of coulomb's law:

Let us consider that two like charges, Q_1 and Q_2, are placed somewhere in space. According to coulomb's law, the electrostatic forces act along the direction of the line joining the two charges. Since we have taken the charges of the same nature, there must be a repulsive force acting between them.

The force on charge Q_1 due to charge Q_2 is $\vec{F}_{12}$. The position vector of charge Q_1 as seen from charge Q_2 is $\vec{r}_{12} = r\hat{r}_{12}$, where r is the separation between the charges.

By applying coulomb's law, we get the following:

$$F = \frac{kQ_1Q_2}{r^2}$$

Also, the position vector is given by,

$$\vec{r}_{12} = r\hat{r}_{12}$$

Therefore, force acting on charge due to charge is given by,

$$\overrightarrow{F_{12}} = \frac{kQ_1Q_2}{r^2}\hat{r}_{12} \text{ .... (i)}$$

The unit position vector is given by,

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$$

By substituting $\hat{r}_{12}$ in equation (i), we get,

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2} \times \frac{\vec{r}_{12}}{r}$$

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2}\vec{r}_{12} \text{ ……….. (ii)}$$

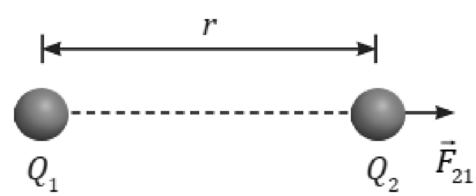
Similarly, the force on charge Q_2 due to charge Q_1 is $\vec{F}_{21}$. The position vector of charge Q_2 as seen from charge Q_1 is, $\vec{r}_{21} = r\hat{r}_{21}$

Therefore, the force acting on charge due to charge is given by,

$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^2}\hat{r}_{21} \text{ … … … … (iii)}$$

The unit position vector is given by,

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$$



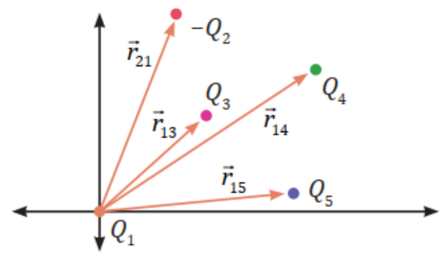
By substituting in $\hat{r}_{12}$ equation (iii), we get,

$$\vec{F}_{12} = \frac{kQ_1Q_2}{r^2} \times \frac{\vec{r}_{12}}{r}$$

$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^3}\vec{r}_{21} \text{ ………... (iv)}$$

- For unlike charges, the forces become attractive in nature and hence, their directions Get changed. Therefore, the force on Q_1 due to Q_2 is expressed as, $\vec{F}_{12} = \frac{kQ_1Q_2}{r^3}\vec{r}_{21}$ and the force on Q_2 due to Q_1 is expressed as, $\vec{F}_{21} = \frac{kQ_1Q_2}{r^3}\vec{r}_{12}$
- Only the formulas for the like charges are enough to remember because to obtain the formulas for the unlike charges, we just need to put the value of charges with sign.
- Since $\vec{r}_{12} = -\vec{r}_{21}$,what we get from equations (i) and (ii) is, $\vec{F}_{12} = -\vec{F}_{21}$. Thus, coulomb's law agrees with newton's third law of motion.

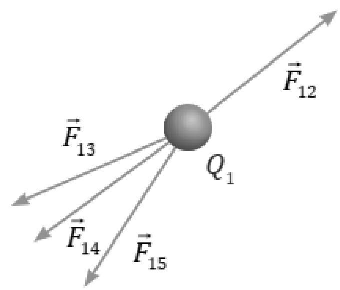
Superposition of coulomb's force:



The resultant electrostatic force on a point will be the vector sum of electrostatic forces due to individual point charges.

For finding the net force on any charge, we have to find the forces by each charge present in the vicinity of it.

Let us consider charge Q_1 and analyse all the forces Acting on it. The forces acting on Q_1 are shown in the Figure.



The net force acting on Q_1 is the vector sum of all the electrostatic force acting on it, which is given by,

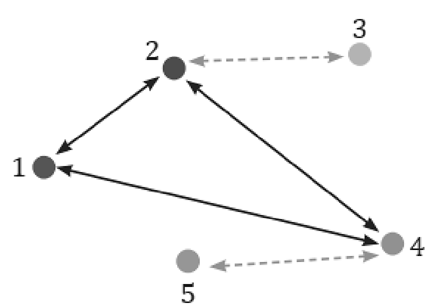
$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$

The force applied by one charge does not affect the force by other charges. They have their individual effects, but the net force acting on the charge changes.

Ex. Five balls numbered 1 to 5 are suspended using separate threads. Pairs (1, 2), (2, 4), and (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion. What should ball 1 therefore be?

(A) Positively charged **(B)** Negatively charged
(C) Neutral **(D)** made of metal

Sol.
Consider five balls as shown in the figure.
The dotted arrow represents the repulsive force between two balls and the solid arrow represents the attractive force between two balls.



It is given that the pairs (1, 2), (2, 4), and (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion.

To show electrostatic repulsion, two charges must be like Charges. Suppose that there is a repulsive force between balls 2 and ball 3, assuming that they have charge A.

Now, for electrostatic attraction to take place between two charges, it is not necessary that both the charges should have opposite charges.

The electrostatic attraction can also happen between a charge particle and a neutral particle (recall the example of the glass rod attracting dry papers). Now, since ball 4 and ball 5 are repelled to each other, they should have the same charge. Therefore, Ball 4 cannot be neutral. On top of this, ball 2 and ball 4 are attracted to each other and since ball 4 cannot be neutral, it should have the opposite charge of type a. Let ball 4 have a charge of type B.

Therefore, ball 1 gets attracted by ball 2 having charge of type A and also gets attracted by ball 4 having charge of type B. Hence, it is possible only if ball 1 is neutral.

Therefore, option (c) is the correct answer.

Ex. Two identical charges in vacuum are separated by a distance of r. The electrostatic force between them is given by F. If 75 % of the charge is taken from one of the charges and given to the other, then the new force becomes F'. Find the ratio $\frac{F}{F\prime}$.

(A) 1 (B) $\frac{16}{9}$

(C) $\frac{16}{7}$ (D) $\frac{7}{16}$

Sol.

Initially, let there be two identical charges of magnitude Q separated by a distance of r. Therefore, according to coulomb's law, the magnitude of the electrostatic force is,

$$F = \frac{kQ^2}{r^2} \text{ .... (i)}$$

Now, if 75 % (or $\frac{3}{4}$) of one charge is given to the other, then those two charges become, $Q_1 = \frac{Q}{4}$ and $Q^2 = \left(Q + \frac{3Q}{4}\right) = \frac{7Q}{4}$

However, the separation between them remains the same.
Therefore, the electrostatic force in between is,

$$F' = \frac{kQ_1Q_2}{r^2}$$

$$F' = \frac{k\left(\frac{Q}{4}\right)\left(\frac{7Q}{4}\right)}{r^2}$$

$$F' = \frac{7}{16}\,\frac{kQ^2}{r^3}$$

$$F' = \frac{7}{16}F$$

$$\frac{F}{F'} = \frac{16}{7}$$

Thus, option (C) is the correct answer.

Ex. Two charge particles, each having charge q and mass m, are apart with a distance of d from each other. If two particles are in equilibrium under the gravitational and electrostatic force, then find the ratio $\frac{q}{m}$.

(A) 10–8 **(B)** 10–10
(C) 108 **(D)** None of these

Sol.

According to the problem, both the particles are in equilibrium under the gravitational and electrostatic force. So, it can be said that the electrostatic force is balanced by the gravitational force. Therefore,

$F_{electrostatic} = F_{gravitational}$

$$\frac{kq^2}{d^2} = \frac{Gm^2}{d^2}$$

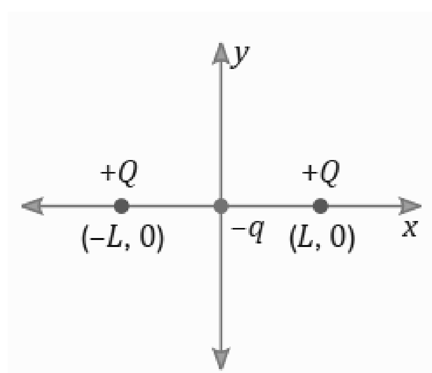
$$\left(\frac{q}{m}\right)^2 = \frac{G}{k}$$

$$\frac{q}{m} = \sqrt{\frac{G}{k}} \approx \sqrt{\frac{10^{-11}}{10^9}}$$

$$\frac{q}{m} \approx \sqrt{10^{-20}} \approx 10^{-10}$$

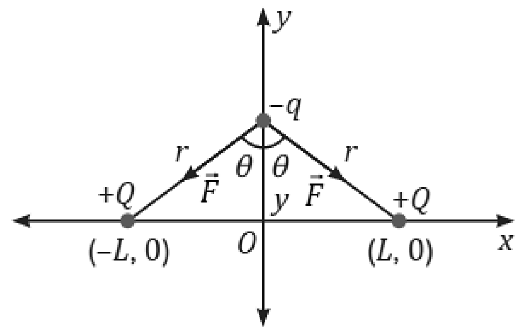
Thus, option (B) is the correct answer

Ex. Two identical point charges +Q are fixed in a gravity-free space at points (L, 0) and (–L, 0). Another particle with mass m and charge –q is placed at the origin. Now, this particle is displaced by a distance of y along the y-axis and then released. Show that this particle will execute oscillatory motion.



Sol.

The only way to prove the particle executes oscillatory motion is to prove that the particle executes SHM because oscillatory motion is a consequence of SHM. Now, if we are able to prove that the force on the particle of charge –q is restoring and is proportional to the displacement, then it will be enough to conclude that the charged particle executes SHM.



Suppose the particle of charge –q is displaced along the y-axis by a distance of y as shown in the figure.

If the force on –q by +Q is given by F, then the net force on the charge –q by both the charges +Q is given by,

$$F_{net} = 2F\cos\theta$$

Also, this force is pointing towards the equilibrium position (O) of the charge –q. Therefore, the net force on the charge –q is restoring in nature.

Now,

$$F_{net} = 2F\cos\theta$$

$$F_{net} = 2\left(\frac{kQq}{r^2}\right)\left(\frac{y}{r}\right)$$

$$F_{net} = 2\left(\frac{kQq}{r^3}\right)y$$

From the figure, it is seen that $r = \sqrt{L^2 + y^2}$

Thus,

$$F_{net} = 2\left(\frac{kQq}{(L^2+y^2)^{\frac{3}{2}}}\right)y$$

Since the charge is displaced slightly along the y-axis, y << L

Hence,

$$F_{net} = 2\left(\frac{kQq}{L^3\left(1+\frac{y^2}{L^2}\right)^{\frac{3}{2}}}\right)y$$

$$F_{net} = 2\left(\frac{kQq}{L^3}\right)y \qquad \text{[Since y << L]}$$

Therefore, the net force on the charge –q is proportional to the displacement of the charge from its equilibrium position and hence, the charge will execute SHM with time period,

$$T = 2\pi\sqrt{\frac{m}{\left(2\left(\frac{kQq}{L^3}\right)\right)}}$$

$$T = 2\pi\sqrt{\frac{mL^3}{2kQq}}$$