ELECTRIC CHARGE AND FIELD
CONTINUOUS CHARGE DISTRIBUTION

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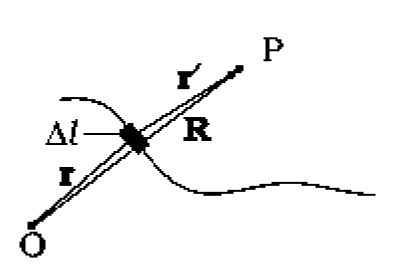
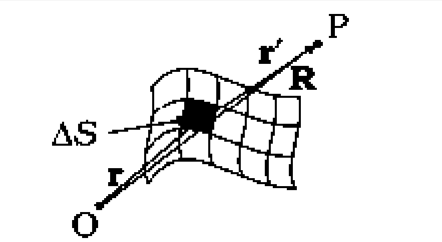
We have so far dealt with charge configurations involving discrete charges q_1, q_2, ..., q_n. One reason why we restricted to discrete charges is that the mathematical treatment is simpler and does not involve calculus. For many purposes, however, it is impractical to work in terms of discrete charges and we need to work with continuous charge distributions. For example, on the surface of a charged conductor, it is impractical to specify the charge distribution in terms of the locations of the microscopic charged constituents. It is more feasible to consider an area element ΔS on the surface of the conductor (which is very small on the macroscopic scale but big enough to include a very large number of electrons) and specify the charge ΔQ on that element. We then define a surface charge density σ at the area element by

$$\sigma = \frac{\Delta Q}{\Delta S}$$



Line charge $\Delta Q = \lambda \Delta l$



Surface charge $\Delta Q = \sigma \Delta S$

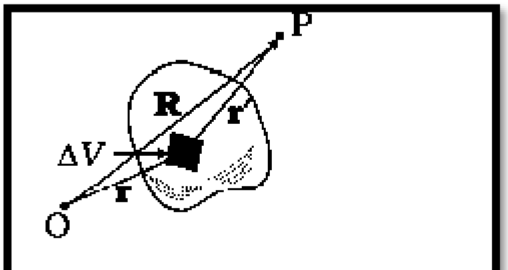
We can do this at different points on the conductor and thus arrive at a continuous function σ, called the surface charge density. The surface charge density σ so defined ignores the quantitation of charge and the discontinuity in charge distribution at the microscopic level*. σ represents macroscopic surface charge density, which in a sense, is a smoothed out average of the microscopic charge density over an area element ΔS which, as said before, is large microscopically but small macroscopically. The units for σ are C/m^2.

Similar considerations apply for a line charge distribution and a volume charge distribution. The linear charge density λ of a wire is defined by

$$\lambda = \frac{\Delta Q}{\Delta I}$$

where Δl is a small line element of wire on the macroscopic scale that, however, includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that line element. The units for λ are C/m. The volume charge density (sometimes simply called charge density) is defined in a similar manner:

$$P = \frac{\Delta Q}{\Delta V}$$



Volume charge $\Delta Q = \rho \Delta V$

where ΔQ is the charge included in the macroscopically small volume element ΔV that includes a large number of microscopic charged constituents. The units for ρ are C/m^3 .
The notion of continuous charge distribution is similar to that we adopt for continuous mass distribution in mechanics. When we refer to
At the microscopic level, charge distribution is discontinuous, because they are discrete charges separated by intervening space where there is no charge.
the density of a liquid, we are referring to its macroscopic density. We regard it as a continuous fluid and ignore its discrete molecular constitution.
The field due to a continuous charge distribution can be obtained in much the same way as for a system of discrete charges, Eq. (1.10). Suppose a continuous charge distribution in space has a charge density ρ. Choose any convenient origin O and let the position vector of any point in the charge distribution be r. The charge density ρ may vary from point to point, i.e., it is a function of r. Divide the charge distribution into small volume elements of size ΔV. The charge in a volume element ΔV is ρΔV.

Now, consider any general point P (inside or outside the distribution) with position vector R . Electric field due to the charge ρΔV is given by Coulomb's law:

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{p\Delta V}{r'^2} r'$$

where r′ is the distance between the charge element and P, and r̂ ′ is a unit vector in the direction from the charge element to P. By the superposition principle, the total electric field due to the charge distribution is obtained by summing over electric fields due to different volume elements:

$$E \cong \frac{1}{4\pi\varepsilon_0}\sum_{all\Delta V}\frac{P\Delta V}{r^2} r'$$

Note that ρ, r′, r̂′ all can vary from point to point. In a strict mathematical method, we should let ΔV→0 and the sum then becomes an integral; but we omit that discussion here, for simplicity. In short, using Coulomb's law and the superposition principle, electric field can be determined for any charge distribution, discrete or continuous or part discrete and part continuous.