APPLICATIONS OF DERIVATIVES

APPROXIMATIONS

Error and Approximation :

Let $y = f(x)$ be a function. If these is an error δx in x then corresponding error in y is

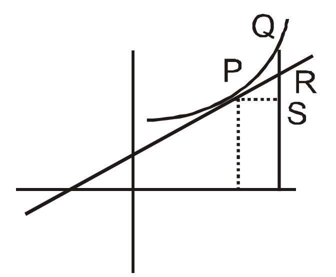
$\delta y = f(x + \delta x) - f(x)$.

We have $\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \frac{dy}{dx} = f'(x)$

We define the differential of y, at point x, corresponding to the increment δx as $f'(x)\,\delta x$ and denote it by dy.

i.e. $dy = f'(x)\,\delta x$.

Let $P(x, f(x))$, $Q((x + \delta x), f(x + \delta x))$ (as shown in figure)



$\delta y = QS$,

$\delta x = PS$,

$dy = RS$

In many practical situations, it is easier to evaluate dy but not δy.

Ex.1 Find the approximate value of $25^{1/3}$.

Sol. Let $y = x^{1/3}$

Let $x = 27$ and $\Delta x = -2$

Now $\Delta y = (x + \Delta x)^{1/3} - x^{1/3} = (25)^{1/3} - 3$

$\frac{dy}{dx}\,\Delta x = 25^{1/3} - 3$

At $x = 27$, $25^{1/3} = 3 - 0.074 = 2.926$

Ex.2 Find the approximate value of $(33)^{1/5}$

Sol. Let $y = f(x) = x^{1/5}$. Now $f'(x) = \frac{1}{5}x^{\frac{-4}{5}}$

We have $f(a + h) = f(a) + h\,f'(a)$

$\Rightarrow (a + h)^{1/5} = a^{1/5} + h.\ a^{-4/5}$

Putting a = 32, h = 1 we get $(33)^{1/5} = 32^{1/5} + 1.\,(32)^{-4/5}$

$= 2 + \frac{1}{5}\,\frac{1}{16} = 2 + \frac{1}{80} = 2.0125$ (approximate)