CONTINUITY AND DIFFERENTIABILITY

DERIVATIVES OF FUNCTIONS IN PARAMETRIC FORMS

Differentiation of Function represented Parametrically :

If x and y are represented as : x = f(t) and y = y(t) (a < t < b), where f(t) and y(t) are differentiable functions and f′(t) ≠ 0, then y defined as a single valued function (continuous) of x is differentiable and its derivative is given b

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\psi'(t)}{\phi'(t)}$$

(t is a parameter)

Remark: $$\frac{d^2y}{dx^2} \neq \frac{\frac{d^2y}{dt^2}}{\frac{dx^2}{dt^2}}$$

Ex.1 Find $\frac{dy}{dx}$ if $y = e^t \cos t, x = e^t \sin t$ (t being a parameter)

Sol. We have $\frac{dy}{dx} = \frac{d}{dt}\left(e^t \cos t\right)$

$$= e^t \cos t - e^t \sin t = e^t(\cos t - \sin t)$$

And $\frac{dx}{dt} = \frac{d}{dt}\left(e^t \sin t\right)$

$$= e^t \sin t + e^t \cos t = e^t(\cos t + \sin t)$$

Then, $\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right) = \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)} = \frac{\cos t - \sin t}{\cos t + \sin t}$