DETERMINANTS

MINORS AND COFACTORS

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Let D be a determinant. Then minor of element a_{ij}, denoted by M_{ij}, is defined as the determinant of the submatrix obtained by deleting i^{th} row & j^{th} column of D. Cofactor of element a_{ij}, denoted by C_{ij}, is defined as

$C_{ij} = (-1)^{i+j} M_{ij}$.

e.g. 1 $D = M_{11} = d = C_{11}$

$M_{12} = c$, $C_{12} = -c$

$M_{21} = b$, $C_{21} = -b$

$M_{22} = a = C_{22}$

e.g.2 $\Delta = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

$M_{11} = \begin{vmatrix} q & r \\ y & z \end{vmatrix} = qz - yr = C_{11}.$

$M_{23} = \begin{vmatrix} a & b \\ x & y \end{vmatrix} = ay - bx, C_{23} = -(ay - bx) = bx -$ ay etc.

Ex.1 Find the minors and cofactors of elements of the matrix $A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$.

Sol. Let M_{ij} and C_{ij} denote respectively the minor and cofactor of element a_{ij} in A. Then,

$M_{11} = \begin{vmatrix} -5 & 6 \\ 5 & 2 \end{vmatrix} = -10 - 30 = -40 \quad \Rightarrow \quad C_{11} = M_{11} = -40$

$M_{12} = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix} = 8 - 18 = -10 \quad \Rightarrow \quad C_{12} = -M_{12} = 10$

$M_{13} = = \begin{vmatrix} 4 & -5 \\ 3 & 5 \end{vmatrix} = 20 + 15 = 35 \quad \Rightarrow \quad C_{13} = M_{13} = 35$

$M_{21} = \begin{vmatrix} 3 & -2 \\ 5 & 2 \end{vmatrix} = 6 + 10 = 16 \quad \Rightarrow \quad C_{21} = -M_{21} = -16$

$M_{22} = = \begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} = 2 + 6 = 8 \quad \Rightarrow \quad C_{22} = M_{22} = 8$

$M_{23} = \begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} = 5 - 9 = -4 \quad \Rightarrow \quad C_{23} = -M_{23} = 4$

Ex.2 Find the minors and cofactors of elements '–3', '5', '–1' & '7' in the determinant

$$\begin{vmatrix} 2 & -3 & 1 \\ 4 & 0 & 5 \\ -1 & 6 & 7 \end{vmatrix}$$

Sol. Minor of $-3 = \begin{vmatrix} 4 & 5 \\ -1 & 7 \end{vmatrix} = 33$; Cofactor of $-3 = -33$

Minor of $5 = \begin{vmatrix} 2 & -3 \\ -1 & 6 \end{vmatrix} = 9$; Cofactor of $5 = -9$

Minor of $-1 = \begin{vmatrix} -3 & 1 \\ 0 & 5 \end{vmatrix} = -15$; Cofactor of $-1 = -15$

Minor of $7 = \begin{vmatrix} 2 & -3 \\ 4 & 0 \end{vmatrix} = 12$; Cofactor of $7 = 12$

Transpose of a Determinant

The transpose of a determinant is the determinant of transpose of the corresponding matrix.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \Rightarrow D^T = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$