DETERMINANTS

INTRODUCTION AND DETERMINANTS

INTRODUCTION

If the equations $a_1x + b_1 = 0$, $a_2x + b_2 = 0$ are satisfied by the same value of x, then

$a_1b_2 - a_2b_1 = 0$. The expression $a_1b_2 - a_2b_1$ is called a determinant of the second order,

and is denoted by : $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

A determinant of second order consists of two rows and two columns.

Next consider the system of equations

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$

If these equations are satisfied by the same values of x and y, then on eliminating x and y we get.

$a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0$

The expression on the left is called a determinant of the third order, and is denoted by

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

A determinant of third order consists of three rows and three columns.

VALUE OF A DETERMINANT

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

Note: Sarrus diagram to get the value of determinant of order three :

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$\begin{array}{ccccc} & & -ve & -ve & -ve \\ a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \\ & & +ve & +ve & +ve \end{array}$$

$= (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)$

Note: that the product of the terms in first bracket

(i.e. $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$) is same as the product of the terms in second bracket.

Ex.1 Evaluate (i) $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix}$

Sol. (i) $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta - \left(-\cos^2\theta\right) = \sin^2\theta + \cos^2\theta = 1.$

(ii) $\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix} = 1 - \log_b a \times \log_a b = 1 - 1 = 0$

$\left[\because \log_b a = \dfrac{1}{\log_a b}\right]$

Ex.2 Eliminate l, m, n from the equations al + cm + bn = 0, cl + bm + an = 0, bl + am + cn = 0 and express the result in the simplest form.

Sol. The given set of equations can also be written as (if $n \neq 0$) :

$$a\left(\frac{\ell}{n}\right) + c\left(\frac{m}{n}\right) + b = 0;$$

$$b\left(\frac{\ell}{n}\right) + a\left(\frac{m}{n}\right) + c = 0$$

$$c\left(\frac{\ell}{n}\right) + b\left(\frac{m}{n}\right) + a = 0;$$

Then, let $\frac{l}{n} = x$; $\frac{m}{n} = y$

System of equations :

$ax + cy + b = 0$ .....**(i)**

$cx + by + a = 0$ .....**(ii)**

$bx + ay + c = 0$ .....**(iii)**

We have to eliminate x & y from these simultaneous linear equations.

Since these equations are satisfied by the same values of x and y, then eliminating x and y we get,

$$\begin{vmatrix} a & c & b \\ c & b & a \\ b & a & c \end{vmatrix} = 0$$

Ex.3 The value of $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$ is -

Sol. $$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix} = 1\begin{vmatrix} 3 & 6 \\ -7 & 9 \end{vmatrix} - 2\begin{vmatrix} -4 & 6 \\ 2 & 9 \end{vmatrix} + 3\begin{vmatrix} -4 & 3 \\ 2 & -7 \end{vmatrix}$$

$= (27 + 42) - 2\,(-36 - 12) + 3\,(28 - 6) = 231$

Alternative : By sarrus diagram

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix} =$$

$$\left|\begin{matrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{matrix}\right|\begin{matrix} 1 & 2 \\ -4 & 3 \\ 2 & -7 \end{matrix}$$

$= (27 + 24 + 84) - (18 - 42 - 72) = 135 - (18 - 114) = 231$