DETERMINANTS

AREA OF TRIANGLE

APPLICATION OF DETERMINANTS IN GEOMETRY

(a) The lines : $a_1x + b_1y + c_1 = 0$ ......(i)

$a_2x + b_2y + c_2 = 0$ ......(ii)

$a_3x + b_3y + c_3 = 0$ ......(iii)

are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

This is the condition for consistency of three simultaneous linear equations in 2 variables.

(b) Equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(c) Area of a triangle whose vertices are (x_r, y_r) ; $r = 1, 2, 3$ is $D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

If $D = 0$ then the three points are collinear.

(d) Equation of a straight line passing through points (x_1, y_1) & (x_2, y_2) is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Ex. 1 Find the area of the triangle whose vertices are (3, 8), (– 4, 2) and (5, 1).

Sol. The area of triangle is given by $\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$$= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$$

$$= \frac{1}{2}(3 + 72 - 14) = \frac{61}{2}$$

Singular & non singular matrix

A square matrix A is said to be singular or non-singular according as |A| is zero or non-zero respectively.

Cofactor matrix & adjoint matrix

Let $A = [a_{ij}]_n$ be a square matrix. The matrix obtained by replacing each element of A by corresponding cofactor is called as cofactor matrix of A, denoted as cofactor A. The transpose of cofactor matrix of A is called as adjoint of A, denoted as adj A.

i.e. if $A = [a_{ij}]_n$

then cofactor $A = [c_{ij}]_n$ when c_{ij} is the cofactor of a_{ij} " i & j.

Adj $A = [d_{ij}]_n$ where $d_{ij} = c_{ji}$ " i & j.