MATRICES

TYPES OF MATRICES

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In this section, we shall discuss different types of matrices.

(i) Column Matrix

A matrix is said to be a column matrix if it has only one column.

For example, $A=\begin{bmatrix} 0 \\ \sqrt{3} \\ -1 \\ 1/2 \end{bmatrix}$ is a column matrix of order 4 × 1.

(ii) Row Matrix

A matrix is said to be a row matrix if it has only one row.

For example, B = $\begin{bmatrix} -\frac{1}{2} & \sqrt{5} & 23 \end{bmatrix}_{1\times 4}$ is a row matrix.

In general, B = $[b_{ij}]_{1 \times n}$ is a row matrix of order 1 × n.

(iii) Square Matrix

A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix. Thus an m × n matrix is said to be a square matrix if m = n and is known as a square matrix of order 'n'.

For example, $A=\begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

In general, A = $[a_{ij}]_{m \times m}$ is a square matrix of order m.

(iv) Diagonal Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix of all the elements, except those in the leading diagonal, are zero i.e., $a_{ij} = 0$ for all $i^{1} j$.

A diagonal matrix of order $n \times n$ having $d_1, d_2, \ldots, d_n$ as diagonal elements is denoted by $\mathrm{diag}[d_1, d_2, \ldots, d_n]$.

For example, the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is a diagonal matrix, to be denoted by

$A = \mathrm{diag}[1, 2, 3]$.

(v) Scalar Matrix

A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$b_{ij} = 0$, when $i \neq j$.

$b_{ij} = k$, when $i = j$, for some constant k.

For example

$$A = [3], B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

are scalar matrices of order 1, 2 and 3, respectively.

(vi) Identity Matrix

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix. In other words, the square matrix $A = [a_{ij}]_{n \times n}$ is an identity matrix, if

$$a_{ij} = \begin{cases} 0 & \text{if} \quad i = j \\ 1 & \text{if} \quad i \neq j \end{cases}.$$

We denote the identity matrix of order n by I_n. When order is clear from the context, we simply write it as I.

For example, $[1], \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3, respectively.

Observe that a scalar matrix is an identity matrix when $k = 1$. But every identity matrix is clearly a scalar matrix.

(vii) Zero Matrix

A matrix is said to be zero matrix or null matrix if all its elements are zero.

For example, $[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [0, 0]$ are all zero matrices. We denote zero matrix by O.

Its order will be clear from the context.

(viii) Upper Triangular Matrix

A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$.

Thus, in an upper triangular matrix, all elements below the main diagonal are zero.

For example, $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 5 \end{bmatrix}$. is an upper triangular matrix.

(ix) Lower Triangular Matrix

A square matrix $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

Thus, in a lower triangular matrix, all elements above the main diagonal are zero.

For example, $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$ is a lower triangular matrix of order 3. A triangular matrix $A = [a_{ij}]\ n \times n$ is called a strictly triangular if $a_{ii} = 0$ for all $i = 1, 2, ..., n$.

EQUALITY OF MATRICES

Definition 2 Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) They are of the same order

(ii) Each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

For example, $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ are equal matrices but $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ are not equal matrices. Symbolically, if two matrices A and B are equal, we write A = B.

Ex.1 Find the values of a, b, c, and d from the following equation.

$$\begin{bmatrix} 2a+b & a-2\,b \\ 5c-d & 4c+3\,d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Sol. By equality of two matrices, equating the corresponding elements, we get

$2a + b = 4 \qquad 5c - d = 11$

$a - 2b = -3 \quad 4c + 3d = 24$

Solving these equations, we get a = 1, b = 2, c = 3 and d = 4

Ex.2 Let $A = \begin{bmatrix} \sin\theta & 1/\sqrt{2} \\ -1/\sqrt{2} & \cos\theta \\ \cos\theta & \tan\theta \end{bmatrix} \& B = \begin{bmatrix} 1/\sqrt{2} & \sin\theta \\ \cos\theta & \cos\theta \\ \cos\theta & -1 \end{bmatrix}$. Find q so that A = B.

Sol. By definition A & B are equal if they have the same order and all the corresponding elements are equal.

Thus we have $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}$ & tan q = – 1

$\Rightarrow$ q = (2n + 1) p – $\frac{\pi}{4}$.