MATRICES

SYMMETRIC AND SKEW SYMMETRIC

SYMMETRIC & SKEW-SYMMETRIC MATRIX

A square matrix A is said to be symmetric if A¢ = A

i.e. Let A = $[a_{ij}]_n$. A is symmetric iff $a_{ij} = a_{ji}$ " i & j.

A square matrix A is said to be skew-symmetric if A¢ = – A

i.e. Let A = $[a_{ij}]_n$. A is skew-symmetric iff $a_{ij} = - a_{ji}$ " i & j.

e.g. $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ is a symmetric matrix.

$B = \begin{bmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{bmatrix}$ is a skew-symmetric matrix.

REMEMBER

(i) In a skew-symmetric matrix all the diagonal elements are zero.

(**Q** $a_{ii} = - a_{ii} \Rightarrow a_{ii} = 0$)

(ii) For any square matrix A, A + A¢ is symmetric & A – A¢ is skew-symmetric.

(iii) Every square matrix can be uniqualy expressed as a sum of two square matrices of which one is symmetric and the other is skew-symmetric.

$A = B + C$, where $B = \frac{1}{2}(A + A')$ & $C = \frac{1}{2}(A - A')$.

Ex.1 If A is symmetric as well as skew symmetric matrix, then A is -

Sol. Let A = $[a_{ij}]$ Since A is skew symmetric $a_{ij} = -a_{ji}$

for i = j, $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$

for i ¹ j, $a_{ij} = -a_{ji}$ [Q A is skew symmetric],

& $a_{ij} = a_{ji}$ [Q A is symmetric]

$a_{ij} = 0$ for all $i^{1} j$

so, $a_{ij} = 0$ for all 'i' and 'j' i.e. A is null matrix.

Ex.2 If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then find the values of q satisfying the euqation $A^T + A = I_2$.

Sol. We have,

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now, $A^T + A = I_2$

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2\cos\theta = 1 \qquad \Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \qquad \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in Z$$

Ex.3 Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Sol. Here $B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

$$P = \frac{1}{2}\left(B + B' \right) = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

Thus $P = \frac{1}{2}\left(B + B' \right)$ is a symmetric matrix.

Also, Let $Q = \frac{1}{2}\left(B - B' \right) = \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{3} \\ -\frac{1}{2} & 0 & -3 \\ -\frac{5}{2} & 3 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}\left(B - B'\right)$ is a skew symmetric matrix.

Now $$P + Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

Thus, B is reresented as the sum of a symmetric and a skew symmetric matrix.