RELATIONS AND FUNCTIONS

CLASSIFICATION OF FUNCTION

FUNCTION :

Let A and B be two given sets and if each element a ∈ A is associated with a unique element b ∈ B under a rule f , then this relation is called function.

Here b, is called the image of a and a is called the pre- image of b under f.

Note :

(i) Every element of A should be associated with B but vice-versa is not essential.

(ii) Every element of A should be associated with a unique (one and only one) element of but any element of B can have two or more relations in A.

1. Representation of Function : It can be done by three methods :

(a) By Mapping

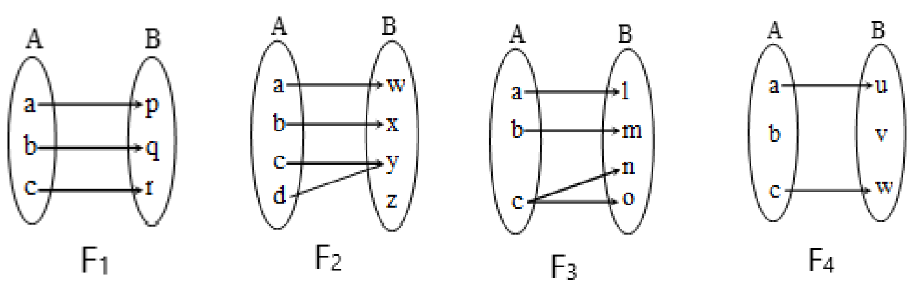
(b) By Algebraic Method

(c) In the form of Ordered pairs

(A) Mapping:

It shows the graphical aspect of the relation of the elements of A with the elements of B .

Ex.



In the above given mappings rule f_1 and f_2

shows a function because each element of A is associated with a unique element of B. Whereas f_3 and f_4 are not function because in f_3, element c is associated with two elements of B, and in f_4 , b is not associated with any element of B, which do not follow the definition of function. In f_2, c and d are associated with same element, still it obeys the rule of definition of function because it does not tell that every element of A should be associated with different elements of B.

(B) Algebraic Method :

It shows the relation between the elements of two sets in the form of two variables x and y where x is independent variable and y is dependent variable.

If A and B be two given sets

$A = \{1,2,3\}$, $B = \{5,7,9\}$

then $f : A \to B$, $y = f(x) = 2x + 3$.

(C) In the form of ordered pairs :

A function $f : A \to B$ can be expressed as a set of ordered pairs in which first element of every ordered pair is a member of A and second element is the member of B. So f is a set of ordered pairs (a, b) such that :

(i) a is an element of A

(ii) b is an element of B

(iii) Two ordered pairs should not have the same first element.

Ex.1 : (i) Which of the following correspondences can be called a function ?

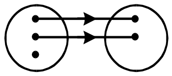
(A) $f(x) = x^3$; $\{-1, 0, 1\} \to \{0, 1, 2, 3\}$

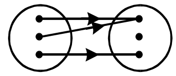
(B) $f(x) = \pm\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

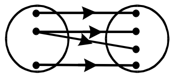
(C) $f(x) = \sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

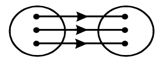
(D) $f(x) = -\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

(ii) Which of the following pictorial diagrams represent the function

(A) 

(B) 

(C) 

(D) 

Sol:

(i) f(x) in (C) and (D) are functions as definition of function is satisfied. while in case of (A) the given relation is not a function, as f(−1)2nd set. Hence definition of function is not satisfied. While in case of (B), the given relation is not a function, as $f(1) = \pm 1$ and $f(4) = \pm 2$ i.e. element 1 as well as 4 in 1st set is related with two elements of 2nd set.Hence definition of function is not satisfied.

(ii) B and D. In (A) one element of domain has no image, while in (C) one element of 1st set has two images in 2nd set

Self practice problems :

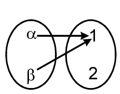
(1) Let g(x) be a function defined on [−1, 1]. If the area of the equilateral triangle with two of its vertices at (0,0) and (x,g(x)) is $\sqrt{3}/4$ sq. unit, then the function g(x) may be.

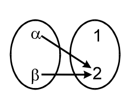
(A) $g(x) = \pm\sqrt{(1-x^2)}$ (B) $g(x) = \sqrt{(1-x^2)}$

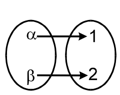
(C) $g(x) = -\sqrt{(1-x^2)}$ (D) $g(x) = \sqrt{(1+x^2)}$

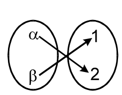
(2) Represent all possible functions defined from {α, β} to {1, 2}.

Ans. (1) B, C

(2) (i) 

(ii) 

(iii) 

(iv) 

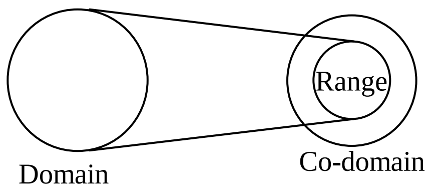
2. Domain, Co-domain and Range :

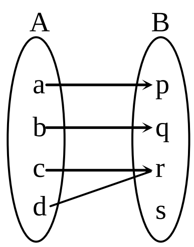
If a function f is defined from a set of A to set B then for f: A→ B set A is called the domain of function f and set B is called the co-domain of function f. The set of the f- images of the elements of A is called the range of function f.

In other words, we can say

Domain = All possible values of x for which f(x) exists.

Range = For all values of x, all possible values of f(x).





Domain = {a,b,c,d} = A

Co-domain = {p,q,r,s} = B

Range = {p,q,r}

Ex.2: Find the domain of following functions :

(i) $f(x) = \sqrt{x^2 - 5}$ (ii) $\sin(x^3 - x)$

Sol: (i) $f(x) = \sqrt{x^2 - 5}$ is real iff $x^2 - 5 \geq 0$

$\Rightarrow |x| \geq \sqrt{5}$ $\Rightarrow x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

the domain of f is $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

(ii) $x^3 - x \in R$

domain is $x \in R$

3. Algebra of functions:

Let f and g be two given functions and their domain are D_f and D_g respectively, then the sum, difference, product and quotient functions are defined as :

(a) $(f + g)(x) = f(x) + g(x), \forall x \in D_f \cap D_g$

(b) $(f - g)(x) = f(x) - g(x), \forall x \in D_f \cap D_g$

(c) $(f . g)(x) = f(x) . g(x), \forall x \in D_f \cap D_g$

(d) $(f/g)(x) = \frac{f(x)}{g(x)}; g(x) \neq 0, \forall x \in D_f \cap D_g$

Ex.3: Which of the following given below is/are a function, from R to R?

(i) $f(x) = x^2$

(ii) f(x) =

(iii) $f(x) = 3x + 4$.

Sol: (i) Yes, because all element of domain (which is R) have images in co-domain (R).

(ii) No, this is not a function because all negative number in a domain (R), do not have images in co-domain.

i.e. $f(-1) =$ (imaginary no.)

$f(-2) =$ (imaginary no.)

(iii) Yes, because all real numbers in domain have images in co-domain.

Note : (i) Any linear expression represents a function.

(ii) Range of ′f′Í co-domain of ¢f¢

(iii) $f : A \rightarrow B$ is not a function, if there is atleast one element in A which does not have a f-image in B or if there is an element in A which has more than one f-images in B.

(iv) A function can also be represented as a set of ordered pairs e.g. f $= \{(1, 2), (2, 3), (3, 4), (4,4)\}$ is a function from $\{1, 2, 3, 4\}$ to $\{2, 3, 4\}$. Clearly f $= \{(1,2), (1, -1), (2, 2), (3, 3)\}$ is not a function as $1 \to 2$ and $1 \to -1$.

Ex.4: Find the domain of following functions :

(i) $f(x) = \sqrt{x+3} - \sqrt{16-x^2}$

(ii) $f(x) = \dfrac{3}{\sqrt{4-x^2}} \log(x^3 - x)$

Sol : (i) $\sqrt{x+3}$ is real iff $x + 3 \geq 0 \Leftrightarrow x \geq -3$

$\sqrt{16-x^2}$ is real iff $16 - x^2 \geq 0 \Leftrightarrow -4 \leq x \leq 4$.

Thus the domain of the given function is

$\{x : x \in [-3, \infty) \cap [-4, 4] = [-3, 4]$

(ii) Domain of $\sqrt{4-x^2}$ is $[-2, 2]$ but $\sqrt{4-x^2} = 0$ for $x = \pm 2$ $\Rightarrow$ $x \in (-2, 2)$

$\log(x^3 - x)$ is defined for $x^3 - x > 0$ i.e. $x(x - 1)(x + 1) > 0$.

$\therefore$ domain of $\log(x^3 - x)$ is $(-1, 0) \cup (1, \infty)$.

Hence the domain of the given function is

$\{(-1, 0) \cup (1, \infty)\} \cap (-2, 2) \equiv (-1, 0) \cup (1, 2)$.

Self practice problems :

(3) Find the domain of following functions.

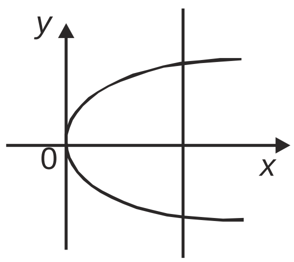
(i) $f(x) = \dfrac{1}{\log(2-x)} + \sqrt{x+1}$

(ii) $f(x) = \sqrt{1-x} - \sin \dfrac{2x-1}{3}$

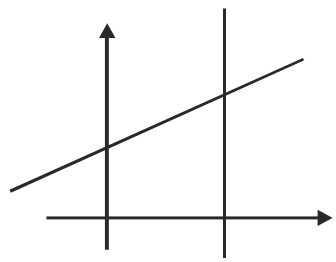
Ans. (i) $[-1, 1) \cup (1, 2)$ (ii) $(-\infty, 1]$

The Graph of a Function

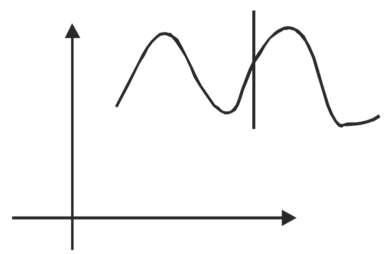
The graph of a function y = f (x) consists of all points (x, f(x)) in the Cartesian plane since by definition of a function, there is exactly one value of y for each x, it follows that no vertical line can intersect the graph of a function of x for twice or more.



not a function of *x*



a function of *x*



a function of *x*

Ex.5: Find the domain of the function $f(x) = \cos^{-1}\left(\frac{|x|-2}{5}\right)$.

Sol: f(x) exist if $-1 \le \frac{|x|-2}{5} \le 1$

⇒ – 5 < | x | – 2 £ 5

⇒ – 3 ≤ | x | £ 7

⇒ | x | ³ – 3 true " x Î R

or | x | ≤ 7 ⇒ x Î [–7, 7].

Ex.6: Find the domain and range of the function $f(x) = \frac{x^2+x+1}{x^2-x+1}$

Sol: x^2 – x + 1 ¹ 0 for any value of x (b^2 – 4ac < 0) so domain of f(x) is R

Range Let f(x) = y

⇒ $\frac{x^2+x+1}{x^2-x+1} = y$

⇒ x^2 (1 – y) + x(1 + y) + (1 – y) = 0

But x is real so b^2 – 4ac ³ 0

⇒ $(1 + y)^2$ – 4 $(1 – y)^2$ ³ 0

$\Rightarrow$ $3y^{2} - 10y + 3 \leq 0$

$\Rightarrow$ $(y - 3)\ (3y - 1) \leq 0$ $\Rightarrow y I \left[\frac{1}{3}, 3\right]$

so range of f(x) $\left[\frac{1}{3}, 3\right]$.

Ex.7: Find the domain of the function f(x), if f(x) = $\sqrt{\boldsymbol{log_{0.5}\, x}}$

Sol: f(x) = $\sqrt{\boldsymbol{log_{0.5}\, x}}$

Now we now that f(x) exist if $\log_{0.5} x\ ^3\ 0$ $x > 0$ (because log x not defined for zero and negative numbers)

$\log_{0.5} x\ ^3\ 0$ $\Rightarrow x \leq (0.5)^\circ$

$\Rightarrow x \leq 1 = x \in (-\infty, 1]$

But $x > 0$

so $x \in (0, 1]$.

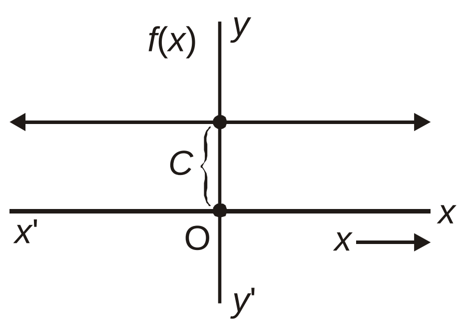
SOME FUNCTIONS AND THEIR GRAPHS

Constant Function

A function denoted by f(x) = C (where C $\in$ R) is known as constant function

Domain = R

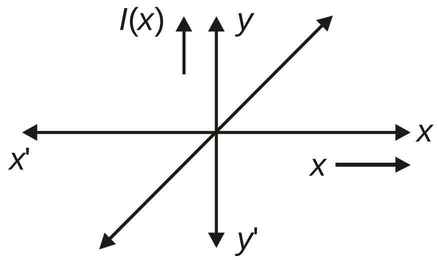
Range = C



Identity Function [I(x)] :

A function which is associated to itself is known as identity function and denoted by I(x) = x

Since x can take any value so domain of this function is R, corresponding value of I(x) is also R, so range is Domain = R , Range = R



Ex.8: Examine whether following pair of functions are identical or not ?

(i) $f(x) = \frac{x^2-1}{x-1}$ and $g(x) = x + 1$

(ii) $f(x) = \sin^2 x + \cos^2 x$ and $g(x) = \sec^2 x - \tan^2 x$

Sol : (i) No, as domain of f(x) is R – {1}

while domain of g(x) is R

(ii) No, as domain are not same. Domain of f(x) is R

while that of g(x) is $R - \left\{(2n+1)\frac{\pi}{2};\ n \in I\right\}$

Modulus Function :

This is also known as absolute value function and denoted by f(x) = |x|

i.e. $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain of this function is set of all real numbers

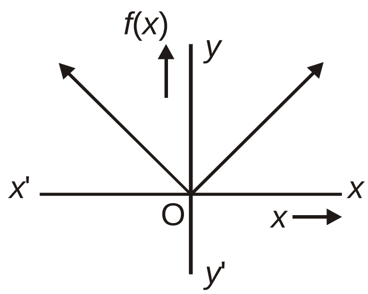
because f(x) exists for all x Î R but |x| ³ 0 so range is all

non-negative real numbers.

Domain = R

Range = [0, ∞]

or $R^{+} \cup \{0\}$



Properties of modulus function :

(a) $|x|^n = |x^n|$

(b) $|x^n| = x^n$, where n is even and n Î z

(c) $|xy| = |x||y|$

(d) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$, (y ¹ 0)

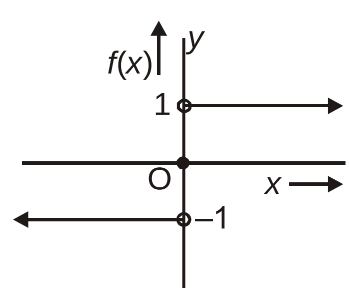
(e) $||x| - |y|| \le |x + y| \le |x| + |y|$

Signum Function

The function f(x), defined as $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

is called signum function. This signum function may also defined as

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$



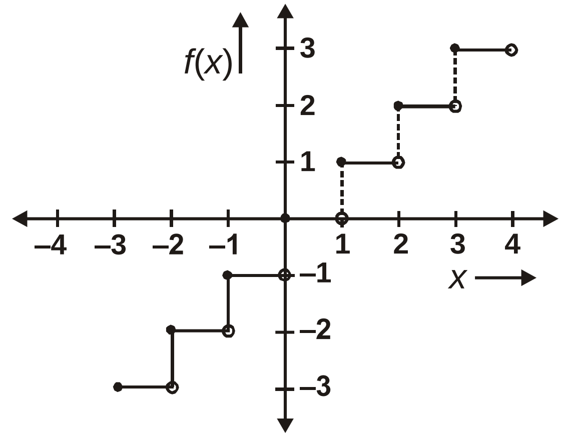
Domain = R

Range = {–1, 0, 1} R

Greatest Integer Function

This function is also known as step function or floor function denoted by f(x) = [x]. By [x] we mean greatest integer less then or equal to x. If n is an integer and x is any real number between n and n + 1

i.e. $n \le x < n + 1$, then $[x] = n$



Thus $[3.4] = 3, [3.99] = 3$

$[-4.99] = -5, [-4.001] = -5$

$[0.3] = 0, [-0.2] = -1$

Domain of [x] is set of all real numbers because [x] exist $x \in R$

But [x] is always integral number so range is set of all integers Z.

Some Properties of [x] :

(a) $[x + k] = [x] + k$, if $k \in Z$

(b) $[-x] = -[x] - 1$

(c) $[x] + [-x] = 0, x \in Z$

(d) $[x] + [-x] = -1, x \notin Z$

(e) $[x] - [-x] = 2x, x \in Z$

(f) $[x] - [-x] = 2[x] + 1, x \notin Z$

(g) $x - 1 < [x] \le x$

(h) $\left\lfloor x+\frac{1}{n}\right\rfloor+\left\lfloor x+\frac{2}{n}\right\rfloor+\left\lfloor x+\frac{3}{n}\right\rfloor+........\left\lfloor x+\frac{n-1}{n}\right\rfloor=[nx]$

(i) $[x + y]\ ^{3}\ [x] + [y]$

(j) $\left\lfloor \frac{(x)}{n} \right\rfloor=\left[\frac{x}{n}\right], n \in N$

(k) $[x]\ ^{3}\ n \Rightarrow x\ ^{3}\ n, n \in z$

(l) $[x] > n \Rightarrow x\ ^{3}\ n + 1, n \in z$

(m) $[x] \le n \Rightarrow x < n + 1, n \in z$

(n) $[x] < n \Rightarrow x < n, n \in z$

Fractional Part Function :

Function denoted by $f(x) = \{x\}$, known as fractional part function.

Also defined as $f(x) = x - [x]$

If $x \in Z$, then $f(x) = 0$ [i.e. $f(2) = 2 - [2] = 0$]

If $x \notin Z$, then f(x) lies between 0 to 1.

i.e. $x \notin Z$, $0 < f(x) < 1$ [i.e. $f(3.4) = 3.4 - [3.4] = 3.4 - 3 = 0.4$]

Note : Fractional part function is a periodic function having period '1'.

Domain = R

Range [0, 1)

Ex.9: Find the value of

$$\left[\frac{1}{3}+\frac{1}{100}\right]+\left[\frac{1}{3}+\frac{2}{100}\right]+\left[\frac{1}{3}+\frac{3}{100}\right]+\ldots\ldots\ldots\left[\frac{1}{3}+\frac{99}{100}\right]$$

Where [×] denote greatest integer function.

Sol: Using properties (h) of step function

Here $x=\frac{1}{3}$, n=100

$$\left[\frac{1}{3}+\frac{1}{100}\right]+\left[\frac{1}{3}+\frac{2}{100}\right]+\left[\frac{1}{3}+\frac{3}{100}\right]+\ldots\ldots\ldots\left[\frac{1}{3}+\frac{99}{100}\right]=\left[\frac{1}{3}\times 100\right]=33$$

Ex.10 Write the equivalent function of the function $f(x) = |x + 2| + |x - 3|$.

Sol: First we find the critical values (values of x where modulus function vanish) which is $x = -2, 3$.

If $x < -2$, then $f(x) = -(x + 2) - (x - 3) = -2x + 1$

If $-2 \le x < 3$, then $f(x) = x + 2 - (x - 3) = 5$

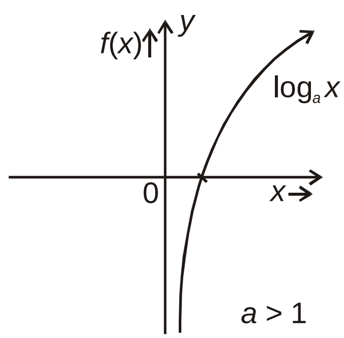
If x^{3} 3, then f(x) = x + 2 + x – 3 = 2x – 1

so $f(x)\begin{cases} -2x+1, & x<-2 \\ 5, & -2\le x<3 \\ 2x-1, & x\ge 3 \end{cases}$

LOGARITHMIC FUNCTION

If f : $R^+ \to R$, f(x) = $\log_a x$, then f(x) is known as logarithmic function

Here f(x) exist if x > 0 and 0 < a < 1 or a > 1 (a ≠ 1)



Properties of logarithmic function

(i) $\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

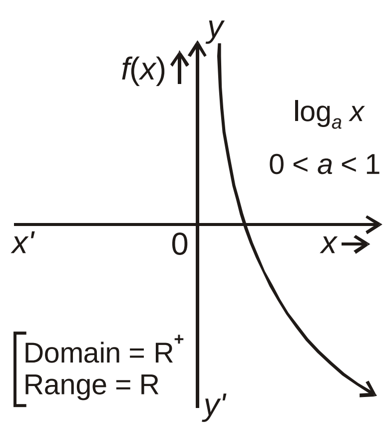
(iii) $\log_a m^n = n \log_a m$

(iv) $\log_{a^q} b^p = \log_a^b$

(v) $\log_a b = \frac{\log_a b}{\log_a a}$ = $\log_x b . \log_a x$

(vi) $\log_a^b . \log_b^a = 1$

(vii) If $\log_a f(x) = y \Rightarrow f(x) = (a)^y$



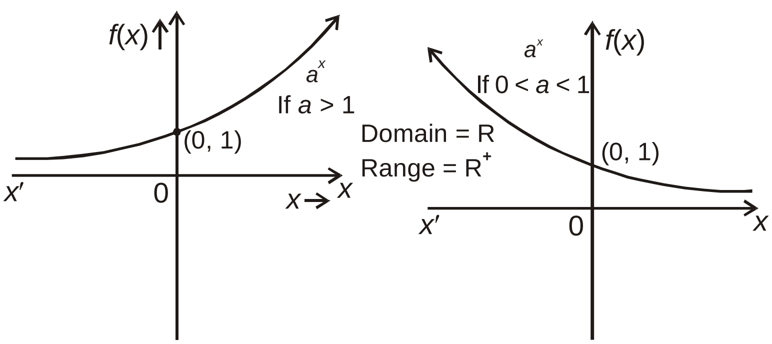
(viii) If $\log_a f(x) \geq \log_a g(x) \Rightarrow \begin{cases} f(x) \geq g(x) \text{ if } a > 1 \\ f(x) \leq g(x) \text{ if } 0 < a < 1 \end{cases}$

(ix) If $\log_a f(x) \geq y \Rightarrow \begin{cases} f(x) \geq (a)^y \text{ if } a > 1 \\ f(x) \leq (a)^y \text{ if } 0 < a < 1 \end{cases}$

(x) If $\log_a f(x) \leq y \Rightarrow \begin{cases} f(x) \leq (a)^y \text{ if } a > 1 \\ f(x) \geq (a)^y \text{ if } 0 < a < 1 \end{cases}$

Exponential Function

$f(x) = a^x$ is known as exponential function $(a > 0)$



Ex.11 How many solutions are there for equation $\log_4 (x - 1) = \log_2 (x - 3)$?

Sol: $\log_4 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_{2^2} (x - 1) = \log_2 (x - 3)$

$\Rightarrow \frac{1}{2} \log_2 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_2 (x - 1)^{\frac{1}{2}} = \log_2 (x - 3)$

$\Rightarrow (x - 1)^{\frac{1}{2}} = (x - 3)$

$\Rightarrow x - 1 = x^2 - 6x + 9$

$\Rightarrow (x - 2)(x - 5) = 0 \quad = x = 2, 5$

But $x - 1 > 0$ and $x - 3 > 0$

$x > 1$ and $x > 3$

So only one solution $x = 5$

KINDS OF FUNCTION

Polynomial Function

The function $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ where $a_0, a_1, a_2, ... a_n$ Î R and n Î N is called a polynomial function of degree n.

Rational Function

A function defined by the quotient of two polynomial function is called rational function for

Example : $\frac{x^2+1}{x^3+x+1}$ is a rational function.

Irrational Function

A function involving one or more radicals of polynomial is called a irrational function

Example : $x^{\frac{3}{2}}+\sqrt{x}+x^2, \frac{x^2+2x+3}{x+\sqrt[3]{x}+5}$ etc.

Algebraic Function

An algebraic function is one which consist of a finite number of terms involving power and roots of the variable x and simple operation, addition, subtraction, multiplication and division i.e. all rational, and irrational functions are algebraic functions.

Transcendental Function

All function which are not algebric are called transcendental function.

Example :

(a) All trigometric function i.e. sin x, cos x etc.

(b) All exponential function, e^x, log x, a^x etc.

(c) Inverse trigonometric function $\sin^{-1} x$, $\cos^{-1} x$, etc.

Note : A transcendental function is not expressed in a finite number of algebraic terms.

Explicit Function

A function in which dependent variable (y) is expressed directly in terms of independent variable (say x)

i.e. $y = x^3 + x^2 + 1$, $y = \frac{x^2+3x+5}{x+2}$, etc.

Implicit Function

A function in which we can't express dependent variable in terms of independent variable.

Example:

$x^3 + y^3 + 3xy = 0$, note that we can't write y or x in terms of x, or y separately.

Even or Odd Function

(a) Even function : If $f(-x) = f(x)$ then f(x) is said to be even function.

Example : $f(x) = \cos x$ is a even function [$f(-x) = \cos(-x) = \cos x = f(x)$]

(b) Odd function : If $f(-x) = -f(x)$ then f(x) is said to odd function.

Example : If $f(x) = x^3 + \tan^3 x$ is a odd function

because $f(-x)= (-x)^3 + [\tan(-x)]^3$

$= - x^3 - \tan^3 x$

$= - [x^3 + \tan^3 x]$

$= - f(x)$

So $f(-x) = - f(x)$

Note : (a) Even function is symmetrical about y-axis while odd function is symmetrical about origin (i.e. in opposite quadrant)

(b) Addition and subtraction of two even function is always even function.

(c) Sum of even and odd function is neither even nor odd function.

(d) Any function 'f' can be represented as the sum of an even and an odd function.

$$f(x)=\frac{1}{2}[f(x)+f(-x)]+\frac{1}{2}[f(n)-f(-x)]$$

where $\frac{1}{2}[f(x)+f(-x)]$ is an even and $\frac{1}{2}[f(n)-f(-x)]$ is an odd function

(e) f(x) = 0 is the only function which is both odd and even.

Ex.12 Is a function f(x) = $x.\frac{e^x+e^{-x}}{e^x-e^{-x}}$ even?

Sol: Yes, f(x) is even, because f(–x) = $(-x)\frac{e^{-x}+e^x}{e^{-x}-e^x}=x\frac{e^x+e^x}{e^x-e^{-x}}$.= f(x)

so f(–x) = f(x).

Ex.13 Show that f(x) is a odd function if $f(x)=\log\left(x^3+\sqrt{1+x^6}\right)$

Sol: $f(x)=\log\left(x^3+\sqrt{1+x^6}\right)$

$$f(-x)=\log\left((-x)^3+\sqrt{1+(-x)^6}\right)$$

= log [–x^3 + $\sqrt{1+x^6}$]

$$=\log\left[-x^3+\sqrt{1+x^6}\times\frac{-x^3+\sqrt{1+x^6}}{-x^3-\sqrt{1+x^6}}\right]$$

$$=\log\left[\frac{x^6-1-x^6}{-x^3-\sqrt{1+x^6}}\right]=\log\left[\frac{-1}{-x^3-\sqrt{1+x^6}}\right]$$

$$f(-x)=\log\frac{1}{x^3+\sqrt{1+x^6}}=-\log\left[x^3+\sqrt{1+x^6}\right]$$

so f(–x) = –f(x) so f(x) is odd function.

Periodic Function

A function 'f' defined on its domain is said to be periodic function if their exist a positive number T such that f(x + T) = f(x) " x ∈ D. Also both x + T and x – T should belong to D. The least value of T, it exists is called, the period of the function.

Ex.14 :

f(x) = sin x

f(x) = sin (x + 2p) = sin (x + 4p) = sin (x + 6p) = ...

Here T = 2p, 4p, 6p ....

Least value of T is 2p, so time period of sin x is 2p

Some Standard Functions and their Period

Function	**Period**
sin x	2p
cos x	2p
tan x	p
{x}	1

Some Special Point about Periodic Function

If period of f(x) is 'T' then

(a) (i) Period of |f(x)| is $\frac{T}{2}$.

(ii) Period of $[f(x)]^n$ is $\frac{T}{2}$, if n is even number ($n \in N$)

(iii) Period of $[f(x)]^n$ is T, if n is odd number ($n \in N$)

(iv) Period of f(ax) and f(ax + b) is $\frac{T}{|a|}$.

(v) Period of $f(\frac{x}{a})$ is |a|.T.

(b) If Period of f(x) and g(x) are same say 'T' then period of f(x) ± g(x) is given by

(i) $\frac{T}{2}$ (if f(x) and g(x) both are even).

(ii) T (if f(x) is any function except even).

(c) If period f(x) is T_1 and g(x) is T_2. Then period of f(x) ± g(x) is given by L.C.M. of T_1 and T_2

(same for $\frac{f(x)}{g(x)}$)

Note : (i) LCM of $\frac{a}{b},\frac{c}{d},\frac{e}{f}=\frac{LCM\ of\ a,c,e}{HCF\ of\ b,d,f}$.

(ii) Sin and sin x^2 is not a periodic function because these can't be written in the form of [f(x + T) = f(x)]

(iii) L.C.M. of rational with irrational is not possible, e.g., L.C.M. of (p, 2, 2p) is not possible as p, 2p ∈ irrational and 2 ∈ rational

Ex.15: Calculate the period of f(x) = sin 3x + cos 2x.

Sol: Period of sin 3x $=\frac{2\pi}{3}$

Period of cos 2x = $\frac{2\pi}{2} = \pi$

So, Period of f(x) is L.C.M. of $\frac{2\pi}{3},\frac{\pi}{1}$ = 2p

Ex.16: If f(x) = $\sqrt{1+sin2x}$ is a periodic function, then find its period.

Sol : f(x) = $\sqrt{1+\sin 2x}$

$= \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x} = \sqrt{(\sin x+\cos x)^2}$

f(x) = |sin x + cos x|

$\left(\text{Remember } \sqrt{x^2} = |x|\right)$

Now period of sin x + cos x is 2p

So, period of |sin x + cos x| is $\frac{2\pi}{2} = \pi$

Bounded and Unbounded Function

f(x) is said to be bounded above, if there exists a fixed number say M such that f(x) is never greater then M for all value of x. Similarly it's bounded below if there exists a fixed number m (say) that f(x) is never less then m

i.e. M ≤ f(x) £ m for all value of x.

f(x) is said to be unbounded if one or both of the upper and lower (M and m) bounds of the function are infinite

Example :

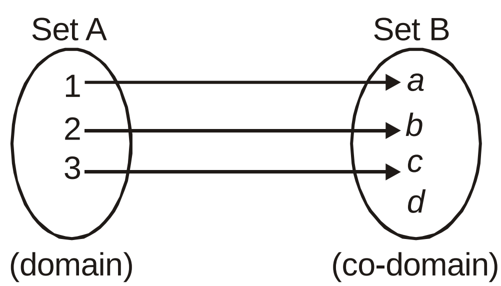
f(x) = 3 + sin x is a bounded function because maximum and minimum value of sin x are +1 and –1

So, $2 \le f(x) \le 4$ for all value of x.

TYPES OF MAPPINGS OR FUNCTIONS

One-one Function or Injective Function :

A function is said to be one-one function if different element in a domain have different images in co-domain.



if $f(x_1) = f(x_2)$ then $x_1 = x_2$
$f(x)$ is one - one function

Note :

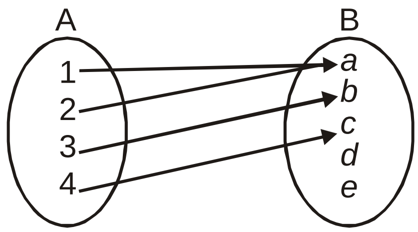
(i) **Example of one-one function :** Linear polynomial function (ax + b), x, e^x, log x, are always one-one functions.

(ii) If $\frac{dy}{dx} > 0$ or $\frac{dy}{dx} < 0$, then y = f(x) is said to be one-one function.

Number of one-one function : If A and B are finite sets having m and n elements respectively, then number of one-one function from A to B = ${}^nP_m$, if n ^{3} m = 0, if n < m.

Many–one Function

A function f : A → B is said to be many one if more than one element in set A have same image in Set B.

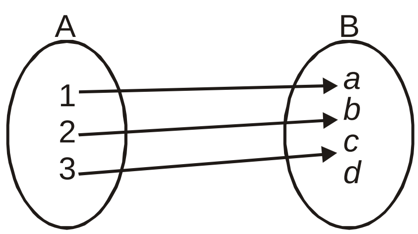


Note : (i) All even function, modulus function, periodic function are always many-one function.

(ii) Square function, Trigonometric function are also many–one function in their domain.

Into Function

A function f: A → B is said to be into function if there exist at least one element in set B having no any pre-image in set B.



In fig set B (co-domain) there is no pre-image, for element d, in set A, so function is into function.

Onto Function

f : A → B, said to be onto function if every element in set B has a pre image in set A.

Range of f = co-domain of f.

Example of Onto function :

log x, linear polynomials, are always onto function.

Possible mappings are

(i) One-one and onto (bijective function)

(ii) Many one and onto

(iii) One-one and into

(iv) Many one-into

Example :

If $f : R \to R$, $f(x) = x^2 + 3x + 2$ then f(x) is many one function.

because $f(x) = x^2 + 3x + 2 = \left(x+\frac{3}{2}\right)^2 - \frac{1}{4}$

$$f(-2) = \left(-2+\frac{3}{2}\right)^2 - \frac{1}{4} = 0$$

$$f(-1) = \left(-1+\frac{3}{2}\right)^2 - \frac{1}{4} = 0$$

So image of –2 and –1 are same f(x) is many one.

Example :

$f(x) = 2x + \sin x$, is one-one because f¢(x) = 2 + cos x, minimum value of cos x is –1.

$f'(x) > 0$ for all $x \in$ Domain = R

Ex.17: (i) Find whether $f(x) = x + \cos x$ is one-one.

(ii) Identify whether the function $f(x) = -x^3 + 3x^2 - 2x + 4$ for $f : R \to R$ is ONTO or INTO

(iii) $f(x) = x^2 - 2x + 3$; $[0, 3] \to A$. Find whether f(x) is injective or not. Also find the set A, if f(x) is surjective.

Sol: (i) The domain of f(x) is R.

$f'(x) = 1 - \sin x$.

$f'(x) \geq 0 \ \forall \ x \in$ complete domain and equality holds at discrete points only

f(x) is strictly increasing on R. Hence f(x) is one-one.

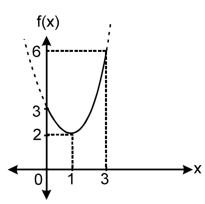
(ii) As range ≡ codomain, therefore given function is ONTO

(iii) $f'(x) = 2(x - 1); 0 \leq x \leq 3$

$$f'(x) = \begin{cases} -ve \ ; \ 0 \leq x < 1 \\ +ve \ ; \ 1 < x < 3 \end{cases}$$

f(x) is non monotonic. Hence it is not injective.

For f(x) to be surjective, A should be equal to its range.



By graph range is [2, 6]

A ≡ [2, 6]