RELATIONS AND FUNCTIONS

BINARY OPERATIONS

DEFINITION OF A BINARY OPERATION

A binary operation can be understood as a function f (x, y) that applies to two elements of the same set S, such that the result will also be an element of the set S. Examples of binary operations are the addition of integers, multiplication of whole numbers, etc. A binary operation is a rule that is applied on two elements of a set and the resultant element also belongs to the same set.

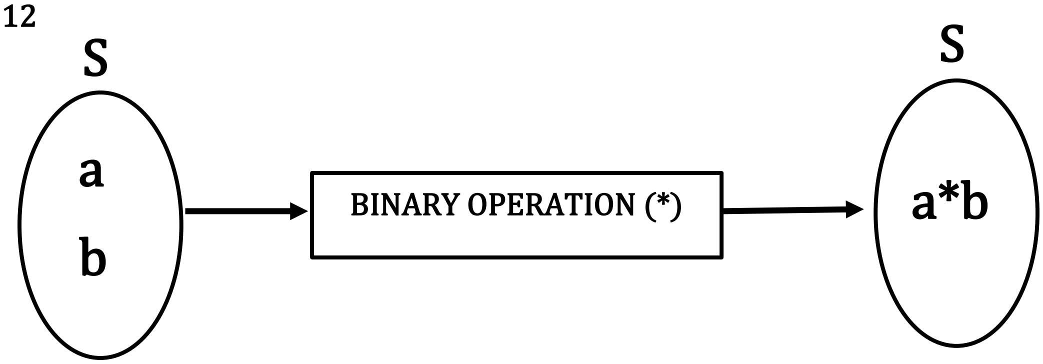
In this article, we will understand the concept of a binary operation, its definition, table, and properties. We will also solve a few examples based on binary operation for a better understanding of the concept.

WHAT IS BINARY OPERATION?

A binary operation on a set is a mapping of elements of the cartesian product set $S \times S$ to S, i.e., $*: S \times S \rightarrow S$ such that $a * b \in S$, for all a, $b \in S$. The two elements of the input and the output belong to the same set S. The binary operation is denoted using different symbols such as addition is denoted by +, multiplication is denoted by $\times$, etc.

BINARY OPERATION DEFINITION

The definition of binary operations states that "If S is a non-empty set, and * is said to be a binary operation on S, then it should satisfy the condition which says, if $a \in S$ and $b \in S$, then $a * b \in S$, $\forall$ a, $b \in S$. In other words, * is a rule for any two elements in the set S where both the input values and the output value should belong to the set S. It is known as binary operations as it is performed on two elements of a set and binary means two.



PROPERTIES OF BINARY OPERATION

Let us learn about the properties of binary operation in this section. The binary operation properties are given below:

- Closure property: An operation * on a non-empty set A has closure property, if a ∈ A, b ∈ A ⇒ a * b ∈ A.

- Additions are the binary operations on each of the sets of Natural numbers (**N**), Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

  The additions on the set of all irrational numbers are not the binary operations.

- Multiplication is a binary operation on each of the sets of Natural numbers (**N**), Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

  Multiplication on the set of all irrational numbers is not a binary operation.

- Subtraction is a binary operation on each of the sets of Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

  Subtraction is not a binary operation on the set of Natural numbers (**N**).

- A division is not a binary operation on the set of Natural numbers (**N**), integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

- Exponential operation $(x, y) \to x^y$ is a binary operation on the set of Natural numbers (**N**) and not on the set of Integers (**Z**).

TYPES OF BINARY OPERATIONS

COMMUTATIVE

A binary operation * on a set A is commutative if a * b = b * a, for all (a, b) ∈ A (non-empty set). Let addition be the operating binary operation for a = 8 and b = 9, a + b = 17 = b + a.

ASSOCIATIVE

The associative property of binary operations hold if, for a non-empty set A, we can write (a * b) *c = a*(b * c). Suppose **N** be the set of natural numbers and multiplication be the binary operation. Let a = 4, b = 5 c = 6. We can write (a × b) × c = 120 = a × (b × c).

DISTRIBUTIVE

Let * and o be two binary operations defined on a non-empty set A. The binary operations are distributive if a*(b o c) = (a * b) o (a * c) or (b o c)*a = (b * a) o (c * a). Consider * to be multiplication and o be subtraction. And a = 2, b = 5, c = 4. Then, a*(b o c) = a × (b − c) = 2 × (5 − 4) = 2. And (a * b) o (a * c) = (a × b) − (a × c) = (2 × 5) − (2 × 4) = 10 − 6 = 2.

IDENTITY

If A be the non-empty set and * be the binary operation on A. An element e is the identity element of a ∈ A, if a * e = a = e * a. If the binary operation is addition(+), e = 0 and for * is multiplication(×), e = 1.

INVERSE

If a binary operation * on a set A which satisfies a * b = b * a = e, for all a, b ∈ A. a^{-1} is invertible if for a * b = b * a= e, a^{-1} = b. 1 is invertible when * is multiplication.

Ex.1: A binary operation table of set S = {a, b, c, d} is given below. Show how does it satisfy the commutative property. Also, find the identity element.

#	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Sol: Given, set S = {a, b, c, d}. In the above table, let us pair up any two elements and see if it satisfies the commutative property of binary operations. Here, we have, a # b = b and b # a = b, b # c = d and c # b = d. So, the given table satisfies the commutative property as x#y=y#x, for all x, y ∈ S.

Now, to find the identity element, we have to find an element e∈S, such that a # e = a = e # a, for all a∈S. From the table, we have, a # b = b = b # a. Also, c # a = c = a # c and a # d = d = d # a. Therefore, a is the identity element of the given binary operation.

Ex.2: Show that addition is a binary operation on natural numbers.

Sol: The set of natural number can be expressed as N = {1, 2, 3, 4, 5, ...}. Every counting number from 1 to infinity comes in the set of natural numbers. So, if we pick up any two elements of this set randomly, let's say 2 and 45, and add those, we get a natural number only. Here, $2 + 45 = 47 \in N$. Therefore, addition is a binary operation on natural numbers.

Ex.3: Show that subtraction is not a binary operation on whole numbers.

Sol: The set of whole numbers can be expressed as W = {0, 1, 2, 3, 4, 5, ...}. Every counting number from 0 to infinity comes in the set of whole numbers. So, if we pick up any two elements of this set randomly, let's say 12 and 45, and subtract those, we may or may not get a whole number. Here, $12 - 45 = -33 \notin W$. Therefore, subtraction is not a binary operation on whole numbers.

Ex.4: Show that division is not a binary operation in N nor subtraction in N.

Sol: Let $a, b \in N$

Case1: Binary operation $* = \text{division}(\div)$

$N \times N \to N$ given by $(a, b) \to (a/b) \notin N$ (as $5/3 \notin N$)

Case2: Binary operation $* = \text{Subtraction}(-)$

$N \times N \to N$ given by $(a, b) \to a - b \notin N$ (as $3 - 2 = 1 \in N$ but $2 - 3 = -1 \notin N$).