HERON'S FORMULA

CONTENTS

➤ SOME BASIC TERMS

◈ **Area :**

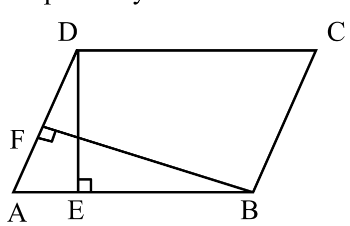
Place, which is covered by base of a body is called area of that body. Area is same from everywhere of the base.

◈ **Height :**

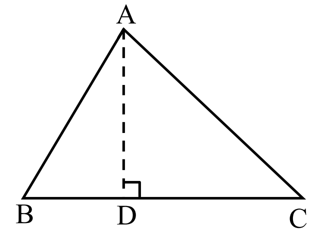
The perpendicular distance is called height and the side having foot of perpendicular, is called base.

❖ EXAMPLES ❖

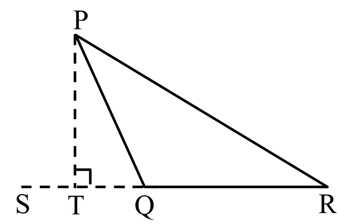
Ex.1 ABCD is a parallelogram. If DE & BF are perpendiculars from D and B on sides AB & DA respectively then their bases are AB and AD respectively.



Ex.2 ABC is an acute angle triangle. AD is height and BC is base



Ex.3 PQR is an obtuse angle triangle at Q. Then height of P from BC is PT but base is QR (not SR).



◈ **Area of triangle =** $\frac{1}{2}$ (base × height) square unit

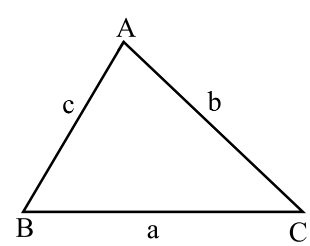
we can use this formula when we can find or given height & base.

◈ **Heron's formula :** If we have all sides of triangle and their is no way to find height then we use this formula for area of triangle.

$$\boxed{\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}}$$

Where s is semi perimeter of Δ.

$$s = \frac{1}{2} \text{ (sum of all sides)} = \frac{1}{2}(a + b + c)$$

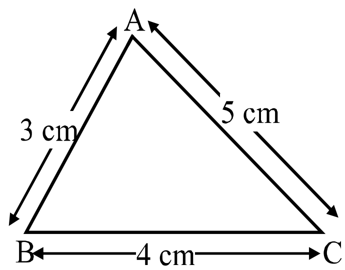


and a, b, c are sides of Δ.

Note : Use $\frac{1}{2}$ (base) (height) for area of right angle triangle, if any two sides are given.

❖ EXAMPLES ❖

Ex.4 For given figure find the s (s – a).



Sol. Perimeter = 2s = 3 + 4 + 5 = 12 cm

$\therefore$ semi perimeter = $\frac{12}{2}$ = 6 cm

$\therefore$ s (s – a) = 6 (6 – 4)

= 6 × 2

= 12 cm.

Ex.5 If semiperimeter of a triangle is 60 cm & its two sides are 45 cm, 40 cm then find third side.

Sol. $\because$ Semiperimeter = 60

$\therefore$ Perimeter = 2 × 60

$\Rightarrow$ Sum of all three sides = 120

(Let third side = x cm)

$\Rightarrow$ x + 45 + 40 = 120

$\Rightarrow$ x + 85 = 120

$\Rightarrow$ x = 120 – 85

$\Rightarrow$ x = 35 cm.

Ex.6 If perimeter of an equilateral triangle is 96 cm, then find its each side.

Sol. $\because$ Length of all sides are equal in equilateral Δ

Let length is x cm

$\therefore$ x + x + x = 96

$\Rightarrow$ 3x = 96

$\Rightarrow$ x = 32 cm.

Ex.7 If one side from two equal sides of a Δ is 14 cm and semiperimeter is 22.5 cm then find the third side.

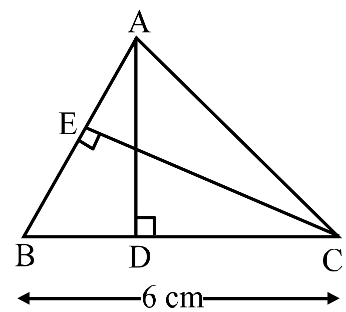
Sol. Let the third side is x cm

$\therefore$ x + 14 + 14 = 2 × 22.5

$\Rightarrow$ x = 45 – 28 = 17 cm.

Ex.8 Find the length of AD in given figure,

if EC = 4 cm and AB = 5 cm.



Sol. $\because$ area of ΔABC = $\frac{1}{2}$(AB × EC)

= $\frac{1}{2}$ (5 × 4) = 10 square cm.

also area of ΔABC = $\frac{1}{2}$(BC × AD)

= $\frac{1}{2}$(6 × AD) = 3AD square cm

$\therefore$ 3AD = 10

$\Rightarrow$ AD = $\frac{10}{3}$ = 3.33 cm.

Note : $\sqrt{2}$ = 1.41, $\sqrt{3}$ = 1.73, $\sqrt{5}$ = 2.23, $\sqrt{6}$ = 2.45, $\sqrt{7}$ = 2.64, $\sqrt{8}$ = 2.82, $\sqrt{11}$ = 3.31, $\sqrt{15}$ = 3.87

Ex.9 Find the area of a triangle whose sides are of lengths 52 cm, 56 cm and 60 cm respectively.

Sol. Let a = 52 cm, b = 56 cm and c = 60 cm.

Perimeter of the triangle = (a + b + c) units

= (52 + 56 + 60) cm = 168 cm

$\therefore \quad s = \frac{1}{2}(a + b + c) = \left(\frac{1}{2} \times 168\right)\text{cm} = 84 \text{ cm}$

(s – a) = (84 – 52) cm = 32 cm,

(s – b) = (84 – 56) cm = 28 cm

and (s – c) = (84 – 60) cm = 24 cm

By Heron's formula, the area of the given triangle is

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{84 \times 32 \times 28 \times 24}\text{cm}^2$

$= \sqrt{14 \times 6 \times 16 \times 2 \times 14 \times 2 \times 6 \times 4}\text{ cm}^2$

$= (14 \times 6 \times 4 \times 2 \times 2)\text{ cm}^2 = 1344 \text{ cm}^2.$

Ex.10 Using Heron's formula, find the area of an equilateral triangle of side a units.

Sol. We have : $s = \frac{1}{2}(a + a + a) = \frac{3a}{2}$

$\therefore \quad (s - a) = \left(\frac{3a}{2} - a\right) = \frac{a}{2},$

$(s - b) = \left(\frac{3a}{2} - a\right) = \frac{a}{2}$

and $(s - c) = \left(\frac{3a}{2} - a\right) = \frac{a}{2}$

So, by Heron's formula, we have :

area = $\sqrt{s(s-a)(s-b)(s-c)}$ sq units

$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}\text{ sq units} = \left(\frac{\sqrt{3}a^2}{4}\right)\text{sq units}$

Hence, area of equilateral triangle of side a is

$\left(\frac{\sqrt{3}a^2}{4}\right)$ sq units.

Note : $\Delta = \left(\frac{\sqrt{3}}{4} \times a^2\right) = \left(\frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}\right)$

$= \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$

$\therefore \quad \text{height} = \frac{\sqrt{3}a}{2}\text{units}$

Ex.11 Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.

Sol. Here, a = 13 cm, b = 13 cm and c = 24 cm.

$\therefore \quad s = \frac{1}{2}(a + b + c) = \frac{1}{2}(13 + 13 + 24)$ cm = 25 cm.

(s – a) = (25 – 13) cm = 12 cm,

(s – b) = (25 – 13) cm = 12 cm

and (s – c) = (25 – 24) cm = 1 cm.

So, by Heron's formula,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25 \times 12 \times 12 \times 1}\text{ cm}^2 = (5 \times 12)\text{cm}^2 = 60 \text{ cm}^2.$

Hence, the area of the given triangle is 60 cm^2.

Ex.12 The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of the triangle.

Sol. $a : b : c = 13 : 12 : 5 \Rightarrow a = 13x, b = 12x \;\&\; c = 5x$

$\therefore$ Perimeter = 450 $\Rightarrow$ 13x + 12x + 5x = 450

$\Rightarrow$ 30x = 450 $\Rightarrow$ x = 15.

So, the sides of the triangle are

a = 13 × 15 = 195 m, b = 12 × 15 = 180 m

and c = 5 × 15 = 75 m

It is given that perimeter = 450

$\Rightarrow 2s = 450$

$\Rightarrow s = 225$

$\therefore \text{ Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{225(225-195)(225-180)(225-75)}$

$\Rightarrow \text{Area} = \sqrt{225 \times 30 \times 45 \times 150}$

$= \sqrt{5^2 \times 3^2 \times 3 \times 5 \times 2 \times 3^2 \times 5 \times 5^2 \times 2 \times 3}$

$\Rightarrow \text{Area} = \sqrt{5^6 \times 3^6 \times 2^2} = 5^3 \times 3^3 \times 2 = 6750 \text{ m}^2.$

Ex.13 Find the percentage increase in the area of a triangle if its each side is doubled.

Sol. Let a, b, c be the sides of the old triangle and s be its semi-perimeter. Then,

$$s = \frac{1}{2}(a+b+c)$$

The sides of the new triangle are 2a, 2b and 2c. Let s' be its semi-perimeter. Then,

$$s' = \frac{1}{2} \times (2a+2b+2c)$$

$$= a + b + c = 2s$$

Let Δ and Δ' be the areas of the old and new triangles respectively. Then,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ and

$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$\Rightarrow \Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$ $[\because s' = 2s]$

$\Rightarrow \Delta' = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

$\therefore$ Increase in the area of the triangle

$= \Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

Hence, percentage increase in area

$= \left(\frac{3\Delta}{\Delta} \times 100\right) = 300\%$

Ex.14 The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find (i) the area of the triangle and (ii) the height corresponding to the longest side.

Sol. Perimeter = 144 cm and ratio of sides = 3 : 4 : 5

Sum of ratio terms = (3 + 4 + 5) = 12.

Let the lengths of the sides be a, b and c respectively.

Then, $a = \left(144 \times \frac{3}{12}\right)$ cm = 36 cm,

$b = \left(144 \times \frac{4}{12}\right)$ cm = 48 cm

and $c = \left(144 \times \frac{5}{12}\right)$ cm = 60 cm.

$\therefore s = \frac{1}{2}(a+b+c) = \frac{1}{2}(36+48+60)$ cm

= 72 cm.

(s – a) = (72 – 36) cm = 36 cm,

(s – b) = (72 – 48) cm = 24 cm

and (s – c) = (72 – 60) cm = 12 cm.

(i) By Heron's formula, the area of the triangle is given by

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{72 \times 36 \times 24 \times 12}\ \text{cm}^2$$

$$= \sqrt{36 \times 36 \times 24 \times 24}\ \text{cm}^2$$

$$= (36 \times 24)\ \text{cm}^2 = 864\ \text{cm}^2.$$

Hence, the area of the given triangle is 864 cm^2.

(ii) Let base = longest side = 60 cm and the corresponding height = h cm.

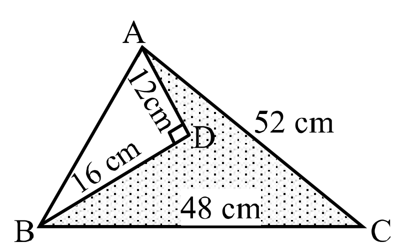
Then, area $= \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$ sq units

$$= \left(\frac{1}{2} \times 60 \times h\right) \text{cm}^2 = (30h)\ \text{cm}^2.$$

$$\therefore 30h = 864 \Rightarrow h = \left(\frac{864}{30}\right) = 28.8.$$

Hence, the height corresponding to the longest side is 28.8 cm.

Ex.15 Find the area of the shaded region in figure :



Sol. By Pythagoras theorem, in ΔADB

$$AB = \sqrt{AD^2 + BD^2}$$

$$= \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400}$$

$AB = 20$ cm.

$\therefore$ area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$\left\{ s = \frac{20+48+52}{2} = 60\,\text{cm} \right\}$$

$$= \sqrt{60(60-20)(60-48)(60-52)}$$

$$= \sqrt{60 \times 40 \times 12 \times 8}$$

$$= \sqrt{(12\times 5)\times(5\times 8)\times 12\times 8}$$

$$= \sqrt{5^2 \times 8^2 \times 12^2} = 5 \times 8 \times 12 = 480 \text{ cm}^2.$$

also area of $\Delta ADB = \frac{1}{2}$ (AD) (BD)

$$= \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$$

$\therefore$ Shaded area = ar(ΔABC) – ar (ΔADB)

$= 480 - 96 = 384$ cm^2.

Ex.16 Find the area of an isosceles triangle of its sides are a cm, a cm and b cm.

Sol. Semi perimeter $= \frac{a+a+b}{2} = \frac{2a+b}{2}$ cm.

$$\therefore\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{2a+b}{2}-a\right)\left(\frac{2a+b}{2}-a\right)\left(\frac{2a+b}{2}-b\right)}$$

$$\Delta = \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2a-b}{2}\right)}$$

$$\Delta = \frac{b}{2\times 2}\sqrt{(2a+b)(2a-b)}$$

$$\Delta = \frac{b}{4}\sqrt{4a^2 - b^2} \text{ square cm.}$$

Ex.17 A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

[NCERT]

Sol. Let 2s be the perimeter of the signal board. Then,

$$2s = a + a + a \Rightarrow s = \frac{3a}{2}$$

Let Δ be the area of the given equilateral triangle. Then,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad \Delta = \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

[$\because$ a = b = c]

$$\Rightarrow \quad \Delta = \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}a^2$$

If, perimeter = 180 cm. Then,

$$2s = 180 \Rightarrow 3a = 180 \Rightarrow a = 60$$

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3} \text{ cm}^2.$$

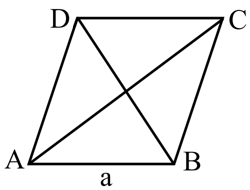
➢ AREA OF QUADRILATERAL

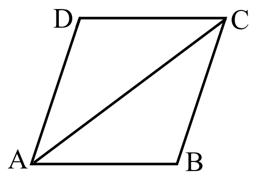
If all four sides and a diagonal are given then by the diagonal we get two triangles. By Heron's formula, we can find area of both triangles and by adding them, we get area of quadrilateral.

➢ AREA OF RHOMBUS

(1) If both diagonals are given (or we can find their length) then area $= \frac{1}{2}$ (Product of diagonals)

(2) If we use Heron's formula then we find area of one triangle made by two sides and a diagonal then twice of this area is area of rhombus.

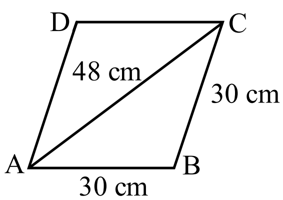




area $= \frac{1}{2}(AC \times DB)$ sq. units   area $= 2(\Delta ABC)$ sq. units

Ex.18 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting ? **[NCERT]**

Sol.



$s = (30 + 30 + 48)/2 = (108)/2 = 54$ m

area $= 2(\Delta ABC)$

$= 2\sqrt{s(s-a)(s-b)(s-c)}$

$= 2\sqrt{54(54-30)(54-30)(54-48)}$

$= 2\sqrt{9 \times 6 \times 24 \times 24 \times 6}$

area $= 2 \times 3 \times 6 \times 24$

$\because$ area required for 18 cows

$= (48 \times 18)$ sq. units $= 864$ sq. units

$\therefore$ area required for 1 cow

$= \frac{48 \times 18}{18} = 48$ sq. units

OR

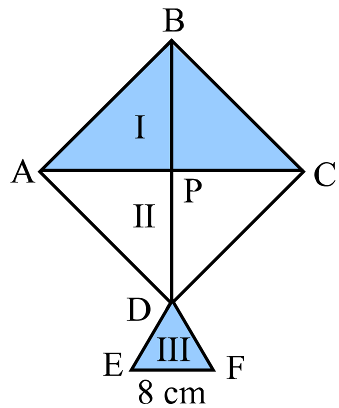
area $= 2\left[\frac{b}{4}\sqrt{4a^2 - b^2}\right]$

$= 2 \times \frac{48}{4}\sqrt{4(30)^2 - (48)^2}$

$= 24\sqrt{3600 - 2304} = 24\sqrt{1296}$

$= 24 \times 36 = 864$ sq. units.

Ex.19 A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it ? **[NCERT]**



Sol. $\because$ ABCD is square

$\therefore$ Both diagonals are equal = 32 cm (each) also diagonals bisect each other at right angle

$\therefore$ AC = 32 cm & BP = PD $= \frac{32}{2} = 16$ cm

$\therefore$ area of (ABC) = ar (ADC)

$= \frac{1}{2}(32) \times 16 = 256 \text{ cm}^2$

and area of ΔDEF

$= \sqrt{s(s-a)(s-b)(s-c)}, \ s = \frac{8+6+6}{2} = 10\text{cm}$

$= \sqrt{10(10-8)(10-6)^2}$

$= 4\sqrt{2 \times 5 \times 2}$

$= 4 \times 2\sqrt{5}$

$= 8\sqrt{5} \text{ cm}^2 = 8 \times 2.236 = 17.88 \text{ cm}^2$

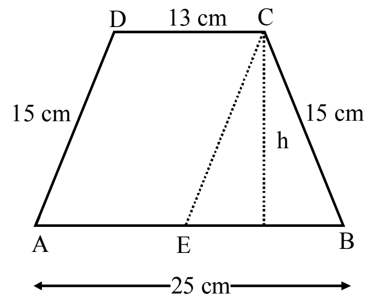
$\therefore$ required areas are 256 cm^2, 256 cm^2, 17.88 cm^2.

➤ AREA OF TRAPEZIUM

Ex.20 Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.

Sol. Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm.

Draw CE || AD.



Now, ADCE is a parallelogram in which AD || CE and AE || CD.

$\therefore$ AE = DC = 13 cm and BE = AB – AE = 25 – 13 = 12 cm

In Δ BCE, we have

$$s = \frac{15+15+12}{2} = 21$$

$\therefore$ Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔBCE

= $\sqrt{21(21-15)(21-15)(21-12)}$

$\Rightarrow$ Area of ΔBCE

= $\sqrt{21\times6\times6\times9} = 18\sqrt{21}\,\text{cm}^2$ ....(i)

Let h be the height of ΔBCE, then

Area of ΔBCE = $\frac{1}{2}$ (Base × Height)

= $\frac{1}{2}$ × 12 × h = 6h .....(ii)

From (i) and (ii), we have,

6h = $8\sqrt{21}$ $\Rightarrow$ h = $3\sqrt{21}$ cm

Clearly, the height of trapezium ABCD is same as that of ΔBCE.

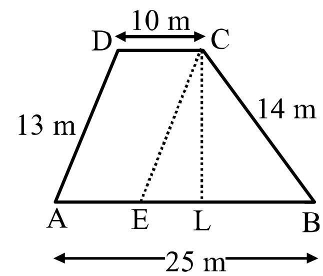
$\therefore$ Area of trapezium = $\frac{1}{2}$ (AB + CD) × h

$\Rightarrow$ Area of trapezium

= $\frac{1}{2}$ (25 + 13) × $3\sqrt{21}$ cm^2 = $57\sqrt{21}$ cm^2.

Ex.21 A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The nonparallel sides are 14 m and 13 m. Find the area of the field. **[NCERT]**

Sol. From C, draw CE || DA. Clearly, ADCE is a parallelogram having AD || CE and DC || AE such that AD = 13 m and DC = 10 m.



$\therefore$ AE = DC = 10 m and CE = AD = 13 m

$\Rightarrow$ BE = AB – AE = (25 – 10) m = 15 m

Thus in BCE, we have

BC = 14 m, CE = 13 m and BE = 15 m

Let s be the semi-perimeter of ΔBCE. Then,

2s = BC + CE + BE = 14 + 13 + 15 = 42

$\Rightarrow$ s = 21

$\therefore$ Area of ΔBCE

$= \sqrt{21\times(21-14)\times(21-13)\times(21-15)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21\times7\times8\times6}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{7^2\times3^2\times4^2} = 84\,\text{m}^2$

Also, Area of ΔBCE = $\frac{1}{2}$ (BE × CL)

$\Rightarrow$ $84 = \frac{1}{2}\times15\times\text{CL}$

$\Rightarrow$ $\text{CL} = \frac{168}{15} = \frac{56}{5}$

$\Rightarrow$ Height of parallelogram ADCE = CL = $\frac{56}{5}$ m

$\therefore$ Area of parallelogram ADCE

= Base × Height = AE × CL = 10 × $\frac{56}{5}$ = 112 m^2

Hence, Area of trapezium ABCD = Area of parallelogram ADCE + Area of ΔBCE

= (112 + 84) m^2 = 196 m^2.

Ex.22 Students of a school staged a rally for cleanliness compaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA, while other through AC, CD and DA (see fig.). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m, and $\angle B = 90°$. Which group cleaned more area and by how much? Find the total area cleaned by the students. **[NCERT]**

Sol. In ΔABC, we have

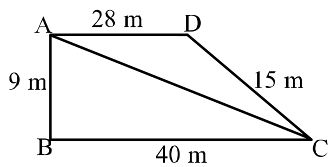
$\angle B = 90°$

$\therefore\ AC^2 = AB^2 + BC^2$

[By Pythagoras Theorem]

$\Rightarrow AC^2 = 9^2 + 40^2 = 1681$

$\Rightarrow AC = 41$



Computation of area of ΔABC :

Δ_1 = Area of $\Delta ABC = \frac{1}{2}(BC \times AB)$

$[\because\ \angle B = 90°]$

$\Rightarrow \Delta_1 = \frac{1}{2}(40 \times 9)\ m^2 = 180\ m^2$

Computation of area of triangle ACD :

Let 2s be the perimeter of ΔACD. Then,

$2s = AC + CD + DA = 41 + 15 + 28 = 84$

$\Rightarrow\ s = 42$ m

$\therefore\ \ \Delta_2$ = Area of ΔACD

$\Rightarrow\ \Delta_2 = \sqrt{s(s-AC)(s-CD)(s-DA)}$

$\Rightarrow\ \Delta_2 = \sqrt{42 \times (42-41) \times (42-15) \times (42-28)}$

$= \sqrt{42 \times 1 \times 27 \times 14} = 126\,m^2$

So, first group cleaned 180 m^2 and second group cleaned 126 m^2

Also, $\Delta_1 + \Delta_2 = (180 + 126)\ m^2 = 306\ m^2$

and, $\Delta_1 - \Delta_2 = (180 - 126)\ m^2 = 54\ m^2$

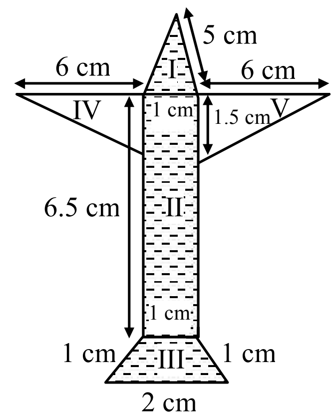
Thus, first group cleaned 54 m^2 more area than the second group and total area cleaned by all the students is 306 m^2.

Ex.23 Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used. **[NCERT]**

Sol. Area of region I;

Region I is enclosed by a triangle of sides

a = 5 cm, b = 5 cm and c = 1 cm



Let 2s be the perimeter of the triangle. Then,

$$2s = 5 + 5 + 1 \Rightarrow\ s = \frac{11}{2}\ cm$$

$\therefore$ Area of region I

$$= \sqrt{s(s-a)(s-b)(s-c)}\ cm^2$$

$\Rightarrow$ Area of region I

$$= \sqrt{\frac{11}{2} \times \left(\frac{11}{2} - 5\right) \times \left(\frac{11}{2} - 5\right) \times \left(\frac{11}{2} - 1\right)}\ cm^2$$

$\Rightarrow$ Area of region I $= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}\ cm^2$

$= \frac{3}{4}\sqrt{11}\ cm^2 = \frac{3}{4} \times 3.32\ cm^2 = 2.49\ cm^2$

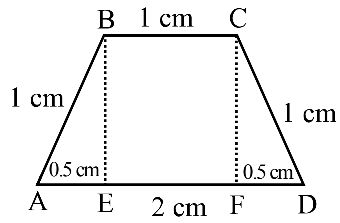
Area of region II :

Region II is a rectangle of length 6.5 cm and breadth 1 cm.

$\therefore$ Area of region II = $6.5 \times 1\ cm^2 = 6.5\ cm^2$

Area of region III :

Region III is an isosceles trapezium as shown in figure.



In ΔABE, we have

$$AB^2 = AE^2 + BE^2$$

$\Rightarrow\ 1 = 0.25 + BE^2$

$\Rightarrow\ BE = \sqrt{0.75} = \sqrt{\frac{3}{4}}$

$\therefore$ Area of region III = $\frac{1}{2}(AD + BC) \times BE$

$= \frac{1}{2}(2 + 1) \times \sqrt{\frac{3}{4}}\ \text{cm}^2 = \frac{3\sqrt{3}}{4}\ \text{cm}^2 = 1.3\ \text{cm}^2$

Area of region IV :

Region IV forms a right triangle whose two sides are of lengths 6 cm and 1.5 cm.

$\therefore$ Area of region IV = $\frac{1}{2} \times 6 \times 1.5\ \text{cm}^2 = 4.5\ \text{cm}^2$

Area of region V :

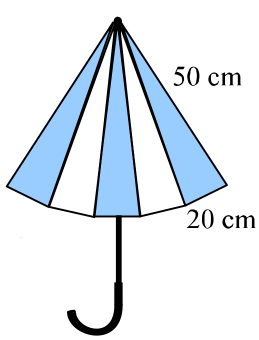
Region IV & V are congruent

$\therefore$ Area of region V = $4.5\ \text{cm}^2$

Hence, total area of the paper used

$= (2.49 + 6.5 + 1.3 + 4.5 + 4.5)\ \text{cm}^2 = 19.29\ \text{cm}^2$

Ex.24 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella? **[NCERT]**



Sol. Sides of one triangular piece of cloth are of lengths a = 20 cm, b = 50 cm and c = 50 cm

Let s be the semi-perimeter of the triangular piece. Then,

$$2s = a + b + c \Rightarrow 2s = 20 + 50 + 50 \Rightarrow s = 60$$

$\therefore\ \Delta$ = Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \Delta = \sqrt{60\times(60-20)\times(60-50)\times(60-50)}\ \text{cm}^2$

$\Rightarrow \Delta = \sqrt{60\times 40\times 10\times 10}\ \text{cm}^2$

$= \sqrt{6\times 4\times 10\times 10\times 10\times 10}\ \text{cm}^2 = 200\sqrt{6}\ \text{cm}^2$

Area of cloth of each colour

$= 5\times 200\sqrt{6}\ \text{cm}^2 = 1000\sqrt{6}\ \text{cm}^2$

Ex.25 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 paisa per cm^2. **[NCERT]**

Sol. Lengths of the sides of the triangular tile are 28 cm, 9 cm and 35 cm.

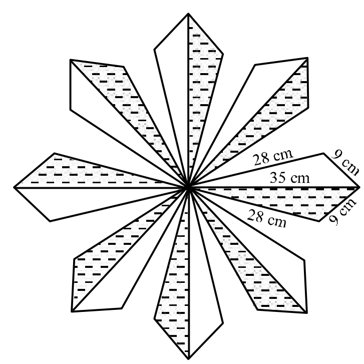
Let s be the semi-perimeter of a tile. Then,

$$s = \frac{28+9+35}{2}\ \text{cm} = 36\ \text{cm}$$

$\therefore$ Area of one tile

$= \sqrt{36\times(36-28)\times(36-9)\times(36-35)}$

$= 36\sqrt{6}\ \text{cm}^2$



So, area of 16 tiles

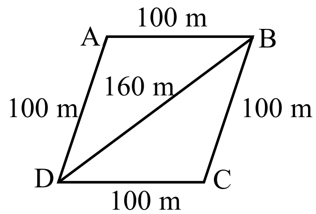
$= 16\times 36\sqrt{6}\ \text{cm}^2 = 576\sqrt{6}\ \text{cm}^2$

Hence, cost of polishing the tiles at the rate of 50 paisa i.e. ₹ $\frac{1}{2}$ per cm^2

$= ₹\ 576\sqrt{6}\times\frac{1}{2} = ₹\ 705.45.$

Ex.26 Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equals parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get ? **[NCERT]**

Sol. Let ABCD be the field which is divided by the diagonal BD = 160 m into two equal parts.



Since ABCD is a rhombus of perimeter 400 m. Therefore,

$$AB = BC = CD = DA = \frac{400}{4} m = 100 \text{ m}$$

Let s be the semi-perimeter of ΔBCD

$$\text{Then, } s = \frac{BC + CD + BD}{2} = \frac{100 + 100 + 160}{2} \text{ m}$$

= 180 m

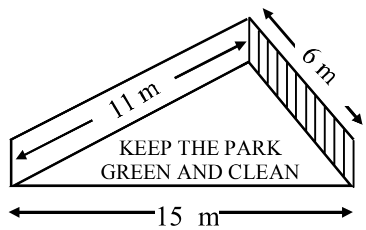
∴ Area of ΔBCD

$$= \sqrt{180 \times (180-100) \times (180-100) \times (180-160)} \text{ m}^2$$

$$= \sqrt{180 \times 80 \times 80 \times 20} \text{ m}^2 = 4800 \text{ m}^2$$

Hence, each of the two children will get an area of 4800 m^2.

Ex.27 There is a slide in a park. One of its side walls has been painted in some colour with a message ''KEEP THE PARK GREEN AND CLEAN'' (see figure). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour. **[NCERT]**



Sol. Clearly, the side wall is in the triangular form with sides a = 15 m, b = 6 m and c = 11 m.

Let 2s be the perimeter of the side wall. Then,

2s = a + b + c ⇒ 2s = 15 + 6 + 11 ⇒ s = 16

∴ s – a = 16 – 15 = 1, s – b = 16 – 6 = 10

and s – c = 16 – 11 = 5

Hence, Area to be painted in colour

= Area of the side wall

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

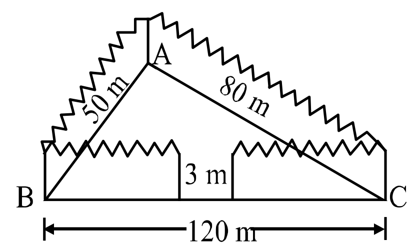
$$= \sqrt{16 \times 1 \times 10 \times 5} = 20\sqrt{2} \text{ m}^2$$

Ex.28 A triangular park ABC has sides 120 m, 80 m and 50 m (see fig.). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of 20 per metre leaving a space 3m wide for a gate on one side. **[NCERT]**

Sol. Computation of area : Clearly, the park is triangular with sides

a = BC = 120 m, b = CA = 80 m and

c = AB = 50 m



It s denotes the semi-perimeter of the park, then

2s = a + b + c ⇒ 2s = 120 + 80 + 50

⇒ s = 125

∴ s – a = 125 – 120 = 5, s – b = 125 – 80 = 45 and s – c = 125 – 50 = 75.

Hence, Area of the park

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{125 \times 5 \times 45 \times 75} \text{ m}^2 = 375\sqrt{15} \text{ m}^2$$

Length of the wire needed for fencing

= Perimeter of the park – width of the gate

= 250 m – 3 m = 247 m

∴ Cost of fencing = ₹ (20 × 247)

= ₹ 4940.

Ex.29 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see fig.). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired both walls for 3 months. How much rent did it pay ? **[NCERT]**

Sol. The lengths of the sides of the walls are 122 m, 22 m and 120 m.

We have,

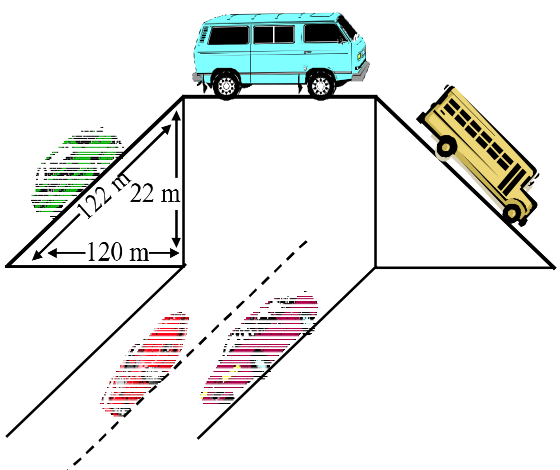
$$122^2 = 120^2 + 20^2$$

So, walls are in the form of right triangles.

$$\therefore \text{ Area of two walls} = 2 \times \left(\frac{1}{2} \times \text{Base} \times \text{Height}\right)$$

$\Rightarrow$ Area of two walls

$$= 2 \times \left(\frac{1}{2} \times 120 \times 22\right) = 2640 \text{ m}^2$$



We have,

$$\text{Yearly rent} = ₹\ 5000 \text{ per m}^2$$

$$\therefore \text{ Monthly rent} = ₹\left(\frac{5000}{12}\right) \text{ per m}^2$$

Hence, rent paid by the company for 3 months

$$= ₹\left(\frac{5000}{12} \times 3 \times 2640\right) = ₹\ 3300000.$$

Ex.30 Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area. **[NCERT]**

Sol. Lets sides of Δ are 12x cm, 17x cm, 25x cm.

$\therefore$ 12x + 17x + 25x = 540 cm

$\Rightarrow$ 54x = 540

$\Rightarrow$ x = 10 cm

$\therefore$ sides are 120 cm, 170 cm, 250 cm

$$\& \ s = \frac{540}{2} = 270 \text{ cm}$$

$$\therefore \text{ area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270(270-120)(270-170)(270-250)}$$

$$= \sqrt{27 \times 10(150)(100)(20)}$$

$$= 10\sqrt{9 \times 3 \times 10 \times (5 \times 3 \times 10)(4 \times 5)}$$

$$= 10 \times 3 \times 2\sqrt{(3 \times 3)(5 \times 5)(10 \times 10)}$$

$= 60 \times 3 \times 5 \times 10 = 9000$ cm^2.

Ex.31 Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm. **[NCERT]**

Sol. Two sides of Δ are 18 cm, 10 cm & Perimeter = 42 cm.

$\therefore$ Third side = 42 – 18 – 10 = 14 cm.

$$s = \frac{42}{2} = 21 \text{ cm}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-18)(21-10)(21-14)}$$

$$= \sqrt{(7 \times 3)(3)(11)(7)}$$

$$= 7 \times 3\sqrt{11} = 21\sqrt{11} \text{ cm}^2 = 21 \times 3.31$$

$= 69.51$ cm^2.