MIXTURE AND ALLIGATION

Alligation

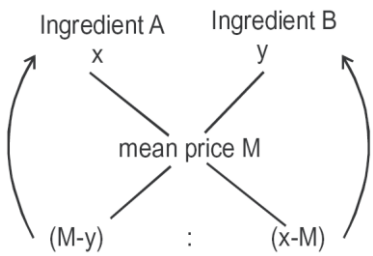
I. **Alligation :** It enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

II. **Mean price :** The cost price of a unit quantity of the mixture is called the mean price.

III. **Basic Formula :** If two ingredients are A and B are mixed of price x and y respectively, then the price of resultant mixture is M mean price. The ratio in which ingredients are mixed is given by rule of alligation

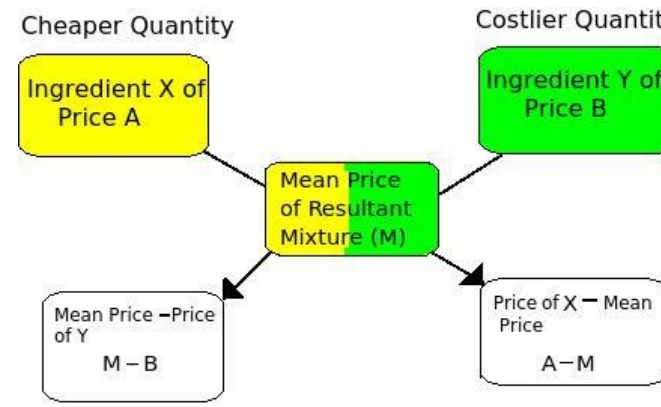
$$R = \frac{M-y}{x-M}$$

The above formula can be respented as



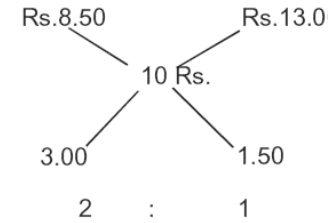
Thus the required ratio is R

$$= \frac{M-y}{x-M} = \frac{y-M}{M-x}$$



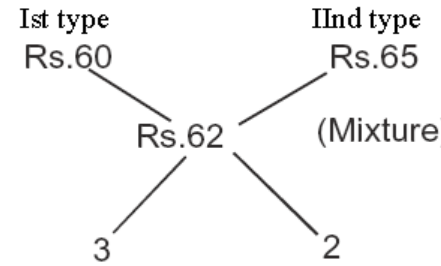
Ex. In what ratio must rice costing Rs.8.50 per kg be mixed with rice costing Rs.13 per kg so that the mixture be worth Rs.10 per kg?

Sol.



Ex. In what ratio must a grocer mix two varities of sugr costing Rs.60 per kg and Rs.65 per kg, so that on selling the mixture at Rs.68.20 per kg he may gain 10%?

Sol. Cost price of 1 kg of mixture



Required Ratio = 3 : 2

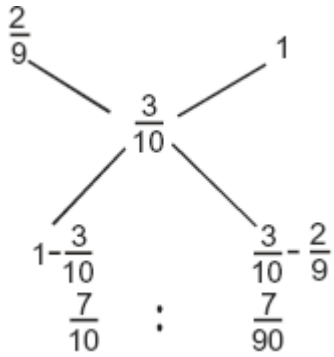
Ex. 729 Litres of mixtures containing milk and water in the ratio 7 : 2. How much more water should be added so that the new mixture contains milk and water in the ratio 7 : 3 ?.

Sol. 729 litre quantity of water = $\frac{2}{9}$

Pure water = 1

Quantity of water in mixture = $\frac{3}{10}$

APPLYING ALLIGATION:



Required Ratio = 9 : 1

9 = 729 litre

$1 = \frac{729}{9} \times 1 = 81$ Litres

IV. Mixture of more than two elements:

These question may seems to be a little tricky at first, but similar concept is applied repeatedly in order to calculate final ratio of ingredients when mixture contains more than two ingredients.

1. Take two ingredients in such away that first ingredient is lower than the mean value and the other one is higher than mean value.

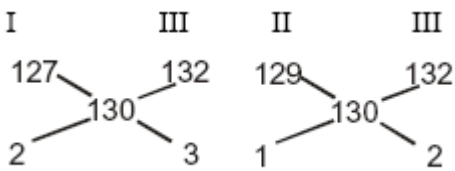
2. Calculate the ratio of ingredients.

3. Repeat for all possible pairs.

4. Final ratio is the ratio obtained from step 2 (if an ingredients is common in the ratios, add value for this particular ingredient)

Ex. Three types of Rice of Rs. 1.27, Rs. 1.29 and Rs. 1.32 per kg are mixed together to be sold at Rs. 1.30 per kg. in what ratio should this rice be mixed.

Sol.

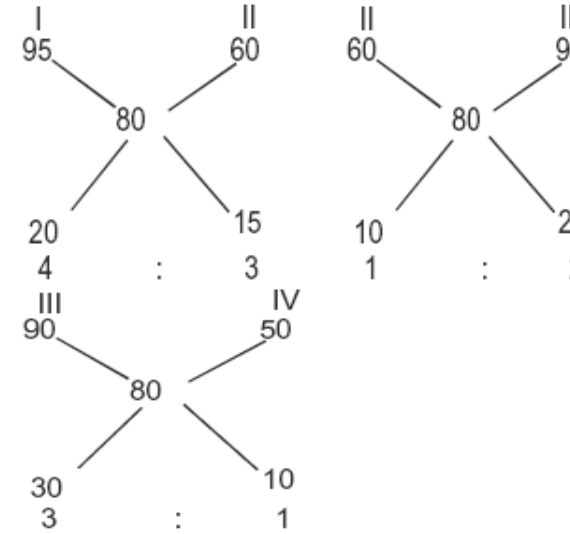


Hence final ratio is 2 : 1 : 3 + 2 = 2 : 1 : 5

V. Mixture containing 4 ingredients:

Ex. How much a shop owner mixture 4 types of rice worth Rs. 95, Rs. 60, Rs. 90 & Rs. 50 perkg so that he can make the mixture of these rice worth Rs. 80 perkg.

Sol.



I : II : III : IV = 4 : 4 : 5 : 1

VI. Concept of replacement:

Suppose a container contains a - units of liquid from which b -units are taken out and replaced by water- after k-operations, the quantity of pure liquid

$$= \left[a\left(1-\frac{b}{a}\right)^k\right] \text{ units.}$$

Ex. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 :65. How much wine did the cask hold originally ?

Sol. Let initial quantity of wine in a cask = x lit

So, After 4 process,

$$4 = \left[x\left(1-\frac{8}{x}\right)^4\right] \text{lit.}$$

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81} \Rightarrow \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4 \Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$$

$\Rightarrow 3x - 24$

$= 2x \Rightarrow x$

$= 24$ lit.

Q.1. In 330 litres of mixtures of milk and water, water is only 24%. The milkman sold 80 litres of this mixture and then he added 60 litres of pure milk and 26 litres of pure water in the remaining mixture. What is the percentage of water in the final mixture?

A39.70%

B25. 59%

C34. 68%

D37. 34%

E16. 89%

Sol: Since, milkman sold 80 litre of mixture

So, remaining mixture

= 330 – 80 = 250 litre

Quantity of water

= 250 × 24/100 = 60 litre

Quantity of milk

= 250 – 60 = 190 litre.

Now, milkman made new mixture in which

water = 60 + 26 = 86 litre

milk = 190 + 60 = 250 litre

Percentage of water in the new mixture

= 86/(86 + 250) × 100

$$= \frac{86}{336} * 100 = 25.59\%$$

Q.2. A paint maker has three different qualities of paint. 806 barrels of 1st quality, 930 barrels of 2nd quality and 992 barrels of 3rd quality. Find the least possible number of buckets of equal size in which different paint of different qualities can be filled without mixing?

(A) 34

(B) 46

(C) 26

(D) 44

(E) None of these

Sol : Oops! Wrong answer!

It is given that paint of, Ist quality = 806 barrels

IInd quality = 930 barrels

IIIrd quality = 992 barrels

Since,

Least number of buckets of equal size will be possible, when we have bottle having highest of largest size.

Largest size having highest or largest size.

Largest size bottle can be found by finding HCF of 806, 930 & 992.

HCF = 2×31 = 62

Therefore, Total numbers of barrels required =13+15+16 = 44

Q.3. Two vessels A and B contain milk and water mixed in the ratio 4 : 3 and 2 : 3. The ratio in which these mixtures be mixed to form a new mixture containing half milk and half water is

(A) 7 : 5

(B) 6 : 5

(C) 5 : 6

(D) 4 : 3

Sol.

Mixture – I Mixture – II

$$\frac{4}{7} \qquad \frac{2}{5}$$

$$\frac{1}{2}$$

$$\frac{1}{2}-\frac{2}{5} \qquad \frac{4}{7}-\frac{1}{2}$$

$$\frac{5-4}{10}=\frac{1}{10} \qquad \frac{8-7}{14}=\frac{1}{14}$$

$\therefore$ required ratio

$$=\frac{1}{10}:\frac{1}{14}=7:5$$

Q.4. A milkman makes 20% profit by selling milk mixed with water at Rs. 9 per litre. If the cost price of 1 litre pure milk is Rs. 10, then the ratio of milk and water in the said mixture is

(A) 3 : 1

(B) 4 : 1

(C) 3 : 2

(D) 4 : 3

So: Short Trick:

SP of Mixture = 9 Rs

120% CP = 9 Rs

CP of Mixture = (9/120)×100 = 7.5 Rs

CP of Milk = 10 Rs

CP of Water = 0 Rs

Using (A) lligation on CP

10 0

7.5

7.5 : 2.5

3 : 1

Basic Method:

Milk : Water = K : 1

$\therefore$ S.P. = (K + 1) × 9

C.P. = 10 K

Gain = 9 – K

Gain

$$\% = \frac{9-K}{10K}\times 100$$

$$\Rightarrow \frac{9-K}{10K}\times 100 = 20$$

$$\Rightarrow 90-10K = 20K$$

$$\Rightarrow 30K = 90 \Rightarrow K = 3$$

Ratio = 3 : 1

Q.5. In two types of stainless steel, the ratio of chromium and steel are 2 : 11 and 5 : 21 respectively. In what proportion should the two types be mixed so that the ratio of chromium to steel in the mixed type becomes 7 : 32?

(A) 2 : 3

(B) 3 : 4

(C) 1 : 2

(D) 1 : 3

Sol: Stainless Steel I II III

Let's take part of Chromium

$$\frac{2}{13} \qquad \frac{5}{26}$$

$$\frac{7}{39}$$

$$\frac{5}{26}-\frac{7}{39} \qquad \frac{7}{39}-\frac{2}{13}$$

$\frac{5}{26}-\frac{7}{39}$ $\frac{7}{39}-\frac{2}{13}$

1/78 : 1/39

$\therefore$ Required ratio = $\frac{1}{78}:\frac{1}{39}=1:2$

Q.6. A vessel has 60 litres of solution of acid and water having 80% acid. How much water be added to make it a solution in which acid forms 60%?

(A) 48 litres

(B) 20 litres

(C) 36 litres

(D) None of these

Sol. In 60 litres of solution,

Water = $\frac{60\times 20}{100}=12$

On adding x litres of water,

$$\frac{12+x}{60+x}\times 100=40$$

$$60+5x=120+2x$$

$$3x=60$$

$x=\frac{60}{3}=20$ litres

Q.7. A mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4 : 5. The quantity of alcohol in the given mixture is

(A) 3 litres

(B) 4 litres

(C) 15 litres

(D) 10 litres

Sol. In original mixture,

Alcohol = 4x litres

Water = 3x litres

On adding 5 litres of water,

$$\frac{4x}{3x+5}=\frac{4}{5}$$

$$\Rightarrow 20x=12x+20$$

$$\Rightarrow 8x=20$$

$$\Rightarrow x=\frac{20}{8}=\frac{5}{2}$$

$\Rightarrow$ *Quantity of alcohol*

$$=4x=4\times\frac{5}{2}$$

$=10$ *litres*

Q.8. A Can contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of mixture are drained off and the Can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the Can initially?

(A) 10

(B) 20

(C) 21

(D) 25

Sol. Suppose the can initially contains 7x and 5x of A and B respectively

Quantity of A in mixture left=7x-(7/12 ×9)

=7x-21/4

Quantity of B in mixture left = 5x-(5/12 ×9)

=5x-15/4

Thus (7x-21/4) ÷ (5x-15/4)+9=7/9

x=3

The can contained 21 litres of A

Q.9. The ratio of spirit and water in two mixtures of 20 litre and 36 litre is 3 : 7 and 7 : 5 respectively. Both the mixtures are mixed together. Now the ratio of the spirit and water in the new mixture is

(A) 25 : 29

(B) 9 : 10

(C) 27 : 29

(D) 27 : 31

Sol. In 20 litres of mixture,

Spirit = $\frac{3}{10} \times 20 = 6$ litre

Water = 14 litre

In 36 litres of mixture

Spirit = $\frac{7}{12} \times 36 = 21$ litre

Water = 15 litre

$\therefore$ Required ratio = (21 + 6) : (14 + 15)

= 27 : 29

Q.10. A litre of pure alcohol is added to 6 litres of 30% alcohol solution. The percentage of water in the solution is

(A) 50%

(B) 65%

(C) 60%

(D) 40%

Sol. In 30% alcohol solution,

Alcohol = $\frac{30}{100} \times 6 = 1.8$ litre

Water = 4.2 litre

On mixing 1 litre of pure alcohol,

Percentage of water = $\frac{4.2}{7} \times 100 = 60\%$

Q.11. In what ratio must a mixture of 30% alcohol strength be mixed with that of 50% alcohol strength so as a mixture of 45% alcohol strength?

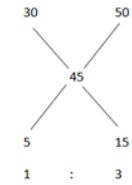
(A) 3:1

(B) 1:3

(C) 2:3

(D) 4:1

Sol.



Hence, the ratio of 30% alcohol: 50% alcohol = 1:3

Q.12. A shopkeeper mixed two varieties of rice at Rs. 20/kg and Rs. 30/kg in the ratio 2 : 3 and sell the mixture at 10% profit. Find the price per kg at which he sold the mixture?

(A) Rs. 26

(B) Rs. 28.8

(C) Rs. 28

(D) Rs. 28.6

Sol. Cost Price of Mixture = (20x2+30x3)/(2+3) = 130/5 = 26 Rs/Kg

Selling Price of Mixture = (26x110)/100 = 28.60 Rs/Kg

Q.14. In a container A, ratio of milk and water is 5:3 & in container B, ratio of milk and water is 11:5. If they are mixed in a ratio of 2:3 in another container, find the ratio of milk and water in the new mixture?

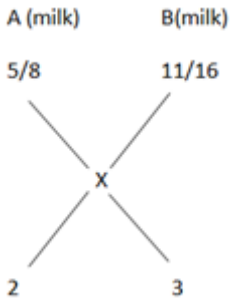
(A) 53:27

(B) 27:53

(C) 53:80

(D) 27:80

Sol.



Hence, the amount of milk in new mixture (X) = (5/8×2 + 11/16×3)/(2+3) = (5/4 +33/16)/ 5
= 53/80
The ratio of milk and water in the new mixture = 53/(80-53) = 53:27

Q.15. A vessel full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :

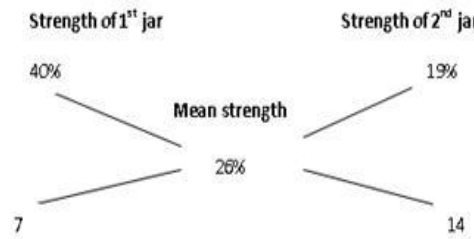
A1/3

B2/3

C2/5

D3/5

Sol.



Ratio of 1st and 2nd quantities= 7:14=1:2
So, Required Quantity replaced = 2/3