DERIVATIVES

1. INTRODUCTION

The rate of change of one quantity with respect to some another quantity has a great importance. For example the rate of change of displacement of a particle with respect to time is called its velocity and the rate of change of velocity is called its acceleration.

The rate of change of a quantity 'y' with respect to another quantity 'x' is called the **derivative** or **differential coefficient** of y with respect to x.

2. DIFFERENTIAL COEFFICIENT

Let y = f(x) be a continuous function of a variable quantity x, where x is independent and y is dependent variable quantity. Let δx be an arbitrary small change in the value of x and δy be the corresponding change in y then $\lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ if it exists, is called the derivative or differential coefficient of y with respect to x and it is denoted by $\frac{dy}{dx}$. y', y_1 or Dy.

So, $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

$$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

The process of finding derivative of a function is called differentiation.

If we again differentiate (dy/dx) with respect to x then the new derivative so obtained is called second derivative of y with respect to x and it is denoted by $\left(\frac{d^2y}{dx^2}\right)$ or y" or y_2 or D^2y.

Similarly, we can find successive derivatives of y which may be denoted by

$$\frac{d^3y}{dx^3}, \frac{d^4y}{dx^4}, \ldots\ldots\ldots, \frac{d^ny}{dx^n}, \ldots\ldots$$

Note : (i) $\frac{\delta y}{\delta x}$ is a ratio of two quantities δy and δx where as $\frac{dy}{dx}$ is not a ratio, it is a single quantity i.e. $\frac{dy}{dx} \neq dy \div dx$

(ii) $\frac{dy}{dx}$ is $\frac{d}{dx}(y)$ in which d/dx is simply a symbol of operation and not 'd' divided by dx.

3. DIFFERENTIAL COEFFICIENT OF SOME STANDARD FUNCTION

The following results can easily be established using the above definition of the derivative–

(i) $\frac{d}{dx}$ (constant) = 0

(ii) $\frac{d}{dx}(ax) = a$

(iii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(iv) $\frac{d}{dx} e^x = e^x$

(v) $\frac{d}{dx}(a^x) = a^x \log_e a$

(vi) $\frac{d}{dx}(\log_e x) = 1/x$

(vii) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$

(viii) $\frac{d}{dx}(\sin x) = \cos x$

(ix) $\frac{d}{dx}(\cos x) = -\sin x$

(x) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xi) $\frac{d}{dx}$ (cot x) = – $\text{cosec}^2 x$

(xii) $\frac{d}{dx}$ (secx)= secx tan x

(xiii) $\frac{d}{dx}$ (cosec x) = – cosec x cot x

(xiv) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$

(xv) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$

(xvi) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(xvii) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(xviii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ $|x| > 1$

(xix) $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

(xx) $\frac{d}{dx}(e^{ax} \sin bx)$

$= e^{ax}(a \sin bx + b \cos bx)$

$= \sqrt{a^2+b^2}\, e^{ax} \sin(bx + \tan^{-1} b/a)$

(xxi) $\frac{d}{dx}(e^{ax} \cos bx) = e^{ax}(a \cos bx - b \sin bx)$

$\sqrt{a^2+b^2}\, e^{ax} \cos(bx + \tan^{-1} b/a)$

4. SOME THEOREMS ON DIFFERENTIATION

Theorem I $\frac{d}{dx}$ [kf(x)]= k d/dx [f(x)], where k is a constant

Theorem II $\frac{d}{dx}[f_1(x) \pm f_2(x) \pm f_3(x) \pm \ldots]$

$= d/dx\,[f_1(x)] \pm d/dx\,[f_2(x)] \ldots\ldots$

Theorem III $\frac{d}{dx}$ [f(x).g(x)]

= f(x) d/dx [g(x)] + g(x) d/dx [f(x)]

Theorem IV

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,d/dx[f(x)] - f(x)\,d/dx\,[g(x)]}{[g(x)]^2}$$

Theorem V Derivative of the function of the function. If 'y' is a function of 't' and t' is a function of 'x' then

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Theorem VI Derivative of parametric equations

If $x = \phi(t)$, $y = \psi(t)$ then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Theorem VII Derivative of a function with respect to another function If f(x) and g(x) are two functions of a variables x, then

$$\frac{d[f(x)]}{d[g(x)]} = \frac{d}{dx}f(x) / \frac{d}{dx}[g(x)]$$

Theorem VIII $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

5. METHODS OF DIFFERENTIATION

5.1 Differentiation of Implicit functions

If in an equation, x and y both occurs together i.e. f(x,y) = 0 and this equation can not be solved either for y or x, then y (or x) is called the implicit function of x(or y).

For example $x^3+y^3+3axy+c = 0$, $x^y+y^x = a^b$ etc.

Working rule for finding the derivative

First Method:

(i) Differentiate every term of f(x,y) = 0 with respect to x.

(ii) Collect the coefficients of dy/dx and obtain the value of dy/dx.

Second Method : If f (x,y) = constant, then

$$\frac{dy}{dx} = \frac{-\partial f/\partial x}{\partial f/\partial y}$$

Where, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are partial differential coefficients of f(x,y) with respect to x and y respectively.

Note : Partial differential coefficient of f (x,y) with respect to x means the ordinary differential coeffcient of f(x,y) with respect to x keeping y constant.

SOLVED PROBLEMS

Ex.1 If $y=(1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$, then find dy/dx

Sol. $y = (1+x^{1/2})(1-x^{1/2}) = 1-x$

$\therefore$ dy/dx = –1

Ex.2 If $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$, then find dy/dx

Sol. $\frac{dx}{d\theta} = a(1+\cos\theta)$, $\frac{dy}{d\theta} = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{1}{2}\theta$

Ex.3 If $y = \log\left(\frac{e^x}{e^x+1}\right)$, then find dy/dx

Sol. $y = \log e^x - \log(e^x+1)$

$= x - \log(e^x+1)$

$\therefore \frac{dy}{dx} = 1 - \frac{e^x}{e^x+1} = \frac{1}{e^x+1}$

Ex.4 If $y = \frac{1}{x^2-a^2}$, then find $\frac{d^2y}{dx^2}$

Sol. $\frac{dy}{dx} = \frac{-2x}{(x^2-a^2)^2} \Rightarrow \frac{d^2y}{dx^2}$

$= -\frac{(x^2-a^2)^2.2 - 2x.2(x^2-a^2).2x}{(x^2-a^2)^4} = \frac{2(3x^2+a^2)}{(x^2-a^2)^3}$

Ex.5 If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$, then find $\frac{dy}{dx}$

Sol. $y = \frac{\sec x-\tan x}{\sec x+\tan x}.\frac{\sec x-\tan x}{\sec x-\tan x}$

$= (\sec x - \tan x)^2/1$

$\therefore \frac{dy}{dx} = 2(\sec x - \tan x)$

$(\sec x\tan x - \sec^2 x)$

$= -2\sec x(\sec x - \tan x)^2$

Ex.6 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then find $\frac{dy}{dx}$

Sol. Let us first express y in terms of x because all alternatives are in terms of x. So

$x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x+y+xy = 0 \quad (\because x \neq y)$

$\Rightarrow y = -\frac{x}{1+x}$

$\therefore \frac{dy}{dx} = -\frac{(1+x)1 - x.1}{(1+x)^2} = -\frac{1}{(1+x)^2}$

Ex.7 If $y = \log_x 10$, then find dy/dx

Sol. $y = \log_x 10 = \frac{\log_e 10}{\log_e x}$

$\therefore \frac{dy}{dx} = \log_e 10\left\{-\frac{1}{(\log_e x)^2}.\frac{1}{x}\right\}$

$= -\frac{1}{x\log_e 10}.\frac{(\log_e 10)^2}{(\log_e x)^2} = -\frac{(\log_x 10)^2}{x\log_e 10}$

Ex.8 If $\cos(xy) = x$, then find $\frac{dy}{dx}$

Sol. $\because \cos(xy) - x = 0$

$\therefore \frac{dy}{dx} = -\frac{-y\sin(xy)-1}{-x\sin(xy)} = -\frac{y+\operatorname{cosec}(xy)}{x}$

Ex.9 If $x^2e^y + 2xye^x + 13 = 0$, then find dy/dx

Sol. Let $f(x,y) = x^2e^y + 2xye^x + 13$

$\therefore \frac{dy}{dx} = -\frac{\partial f}{\partial x}/\frac{\partial f}{\partial y} = -\frac{2xe^y+2ye^x+2xye^x}{x^2e^y+2xe^x}$

Dividing Num^r and Den^r by e^x

$\frac{dy}{dx} = -\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$

Ex.10 If $x^y y^x = 1$, then find $\frac{dy}{dx}$

Sol. Taking log on both sides, we have

$y\log x + x\log y = 0$

Now using partial derivatives, we have

$\frac{dy}{dx} = -\frac{y/x+\log y}{\log x + x/y} = -\frac{y(y+x\log y)}{x(x+y\log x)}$

Ex.11 If $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$, then find dy/dx

Sol. $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$

Taking logarithm of both the sides, we get

$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$

$\Rightarrow y = x^2 + x^2\tan(\log x)$

$\therefore$ $dy/dx = 2x + 2x$

$\tan(\log x) + x^2\sec^2(\log x).\frac{1}{x}$

$= 2x[1+\tan(\log x)] + x\sec^2(\log x).$

Ex.12 If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, then find dy/dx

Sol. Substituting $x = \sin\theta$ and $y = \sin\phi$ in the given equation, we get

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos\frac{\theta+\phi}{2}.\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}.\sin\frac{\theta-\phi}{2}$

$\Rightarrow \cot \dfrac{\theta-\phi}{2} = a \;\Rightarrow\; \theta - \phi = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiating with respect to X, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

Ex.13 If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$ then find dy/dx

Sol. Here $y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

$$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Ex.14 If $e^{x+e^{x+e^{x+.....\infty}}}$, then find dy/dx

Sol. $y = e^{x+y}$

$\Rightarrow \log y = x + y$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$$

Ex.15 If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then find $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

Sol. $\dfrac{dy}{dx} = \left(\dfrac{dy}{d\theta}\right) / \left(\dfrac{dx}{d\theta}\right)$

$$= \frac{3a\sin^2\theta, \cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

$\therefore$ exp. $= \sqrt{1+\tan^2\theta} = \sec\theta$

Ex.16 If $(a+bx)e^{y/x} = x$, then find $x^3\dfrac{d^2y}{dx^2}$

Sol. Taking logarithm of both the sides

$\log(a+bx) + y/x = \log x$

Now differentiating with respect to x, we get

$$\frac{b}{a+bx} + \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} - y = x^2\left(\frac{a+bx-bx}{x(a+bx)}\right) = \frac{ax}{(a+bx)}$$

Again differentiating with respect to x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - ax(b)}{(a+bx)^2}$$

$$x^3\frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2 = \left(x\frac{dy}{dx} - y\right)^2$$

Ex.17 Find the derivatives of the following functions from first principle :

(i) $(x-1)(x-2)$ (ii) $x^3 - 27$.

Sol. (i) Let $f(x) = (x-1)(x-2) = x^2 - 3x + 2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{[(x+h)^2 - 3(x+h) + 2] - [x^2 - 3x + 2]}{h}$$

$$= \lim_{h\to 0}\frac{h^2 + 2xh - 3h}{h} = \lim_{h\to 0} h + 2x - 3$$

$= 0 + 2x - 3 = 2x - 3$

(ii) $f(x) = x^3 - 27$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{[(x+h)^3 - 27] - (x^3 - 27)}{h}$$

$$= \lim_{h\to 0}\frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h\to 0}(3x^2 + 3xh + h^2)$$

$= 3x^2 + 3x \times 0 + 0^2 = 3x^2$

18. Find the derivatives of the following fucntions from first principle.

(i) $\sin(x+1)$ (ii) $\cos\left(x - \dfrac{\pi}{8}\right)$

Sol. (i) Let $f(x) = \sin(x+1)$

$$\therefore f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\left(\frac{(x+h+1+x+1)}{2}\right).\sin\left(\frac{x+h+1-x-1}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\left(x+1+\frac{h}{2}\right)\sin\frac{h}{2}}{h} = \lim_{h\to 0}\cos\left(x+1+\frac{h}{2}\right).\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$= \cos(x+1).\ 1 = \cos(x+1)$

(ii) Let $f(x) = \cos \lim_{h\to 0}\left(x - \dfrac{\pi}{8}\right)$

$$\therefore f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos\left(x+h-\frac{\pi}{8}\right) - \cos\left(x-\frac{\pi}{8}\right)}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin\left[\frac{x+h-\frac{\pi}{8}+x-\frac{\pi}{8}}{2}\right].\sin\left[\frac{x+h-\frac{\pi}{8}-x+\frac{\pi}{8}}{2}\right]}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= -\lim_{h\to 0}\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right).\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -\sin\left(x-\frac{\pi}{8}\right).\ 1 = -\sin\left(x-\frac{\pi}{8}\right)$$

EXERCISE

Q.1 Differentiate the following functions :

(i) x^{-3} (ii) $\frac{1}{x}$ (iii) $\frac{1}{5x}$

(iv) $6x^5 + 4x^3 - 3x^2 + 2x - 7$

(v) $4x^3 + 3 \cdot 2^x + 6 \cdot \sqrt[8]{x^{-4}} + 5\cot x$

(vi) $4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\operatorname{cosec} x} + 9$

(vii) $(2x+3)(3x-5)$ (viii) $\frac{3x^2+4x-5}{x}$

Q.2 (i) If $y = 6x^5 - 4x^4 - 2x^2 + 5x - 9$, find $\frac{dy}{dx}$ at $x = -1$.

(ii) If $y = (\sin x + \tan x)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

(iii) If $y = \frac{(2-3\cos x)}{\sin x}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

Q.3 If $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, show that $2x.\frac{dy}{dx} + y = 2\sqrt{x}$.

Q.4 If $y = \left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right)$, prove that

$(2xy)\left(\frac{dy}{dx}\right) = \left(\frac{x}{a} - \frac{a}{x}\right)$.

Q.5 If $y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$, find $\frac{dy}{dx}$.

Q.6 $y = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$, find $\frac{dy}{dx}$.

Q.7 Find the derivative of each of the following from the first principle :

(i) $x^3 - 2x^2 + x + 3$ (ii) x^8 (iii) $\frac{1}{x^3}$

(iv) $\sqrt{5x-4}$ (v) $\frac{1}{\sqrt{x+2}}$ (vi) $\frac{x^2+1}{x}, x \neq 0$

(vii) $\sqrt{\cos 3x}$ (viii) $\sqrt{\sec x}$ (ix) $\tan^2 x$

(x) $\sin(2x+3)$ (xi) $\tan(3x+1)$

Q.8 Differentiate:

(i) $x^2\sin x$ (ii) $e^x\cos x$ (iii) $x^4\tan x$

(iv) $(3x-5)(4x^2-3+e^x)$ (v) $(x^2 - 4x + 5)(x^3 - 2)$

(vi) $(x^2 + 2x - 3)(x^2 + 7x + 5)$

Q.9 Differentiate :

(i) $\frac{e^x \sin x}{\sec x}$ (ii) $\frac{2^x \cot x}{\sqrt{x}}$ (iii) $\frac{e^x(x-1)}{(x+1)}$

(iv) $\frac{x\tan x}{(\sec x + \tan x)}$ (v) $\left(\frac{\sin x - x\cos x}{x\sin x + \cos x}\right)$

Differentiate the following with respect to x: from Q.10 to Q.22 :

Q.10 $\cos(\sin\sqrt{ax+b})$ **Q.11** $e^{2x}\sin 3x$ **Q.12** $e^{3x}\cos 2x$

Q.13 $e^{-5x}\cot 4x$ **Q.14** $\cos(x^3 . e^x)$

Q.15 $e^{(x\sin x + \cos x)}$ **Q.16** $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

Q.17 $\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$ **Q.18** $\sqrt{\frac{1-x^2}{1+x^2}}$

Q.19 $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ **Q.20** $\sqrt{\frac{1+\sin x}{1-\sin x}}$

Q.21 $\sqrt{\frac{1+e^x}{1-e^x}}$ **Q.22** $\frac{e^{2x}+x^3}{\operatorname{cosec} 2x}$

Find $\frac{dy}{dx}$ from Q.23 to Q.27 :

Q.23 $y = \sin(\sqrt{\sin x + \cos x})$ **Q.24** $y = e^x \log(\sin 2x)$

Q.25 $y = \cos\left(\frac{1-x^2}{1+x^2}\right)$ **Q.26** $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

Q.27 $y = \frac{\sin x + x^2}{\cot 2x}$

Q.28 If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, show that $\frac{dy}{dx} + y^2 + 1 = 0$.

Q.29 If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, show that $\frac{dy}{dx} = \sec^2(x + \pi/4)$

Q.30 If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx} + y = 0$.

Q.31 If $y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$, show that $\frac{dy}{dx} = \sec x(\tan x + \sec x)$.

ANSWER KEY

1. (i) $\frac{-3}{x^4}$ (ii) $\frac{-1}{x^2}$ (iii) $\frac{-1}{5x^2}$ (iv) $30x^4+12x^2-6x+2$ (v) $12x^2+(3\log 2).2^x-3x^{-3/2}-5\text{cosec}^2x$

(vi) $-4\text{cosec}^2x+\frac{1}{2}\sin x+2\sec x\tan x+3\text{cosec}x\cot x-6\sin x$ (vii) $12x-1$ (viii) $\left(3+\frac{5}{x^2}\right)$

2. (i)55(ii)9/2(iii) $2(3-\sqrt{2})$ **5.** $-\text{cosec}^2x$ **6.** $-\sin x$

7. (i) $3x^2-4x+1$ **(ii)** $8x^7$ **(iii)** $\frac{-3}{x^4}$ **(iv)** $\frac{5}{2\sqrt{5x-4}}$ **(v)** $\frac{-1}{2(x+2)^{3/2}}$ **(vi).** $\left(1-\frac{1}{x^2}\right)$ **(vii)** $\frac{-3}{2}\cdot\frac{\sin 3x}{\sqrt{\cos 3x}}$ (viii) $\frac{1}{2}\tan x\sqrt{\sec x}$ **(ix).** $2\tan x\sec^2 x$ **(x)** $2\cos(2x+3)$ **(xi).** $3\sec^2(3x+1)$

8. **(i)** $(x^2\cos x+2x\sin x)$ **(ii)** $e^x(\cos x-\sin x)$ **(iii)** $x^4\sec^2x+4x^3\tan x$
(iv) $(36x^2-40x+3xe^x-2e^x-9)$ **(v)** $5x^4-16x^3+15x^2-4x+8$ **(vi)** $4x^3+27x^2+32x-11$

9. **(i)** $e^x[\cos 2x+\sin x\cos x]$ **(ii)** $\frac{2^x[-x\,\text{cosec}^2x+x(\log 2)\cot x-\frac{1}{2}\log x]}{x^{3/2}}$

(iii) $\frac{e^x(x^2+1)}{(x+1)^2}$ **(iv)** $\frac{x\sec x(\sec x-\tan x)+\tan x}{(\sec x+\tan x)}$ **(v)** $\frac{x^2}{(x\sin x+\cos x)^2}$

10. $\frac{-a}{2\sqrt{ax+b}}(\cos\sqrt{ax+b}).\sin(\sin\sqrt{ax+b})$ **11.** $e^{2x}(3\cos 3x+2\sin 3x)$

12. $e^{3x}(3\cos 2x-2\sin 2x)$ **13.** $-e^{5x}(4\text{cosec}^2 4x+5\cot 4x)$ **14.** $-x^2e^x(x+3).\sin(x^3e^x)$

15. $(x\cos x).e^{(x\sin x+\cos x)}$ **16.** $\frac{-4}{(e^x-e^{-x})^2}$ **17.** $\frac{-8}{(e^{2x}-e^{-2x})^2}$ **18.** $\frac{-2x}{(1+x^2)^{3/2}(1-x^2)^{1/2}}$

19. $\frac{-2a^2x}{(a^2+x^2)^{3/2}(a^2-x^2)^{1/2}}$ **20.** $\frac{1}{(1-\sin x)}$ or $\sec x(\sec x+\tan x)$ **21.** $\frac{e^x}{(1-e^x)(1-e^{2x})^{1/2}}$

22. $2e^{2x}(\cos 2x+\sin 2x)+2x^3\cos 2x+3x^2\sin 2x$ **23.** $\frac{(\cos x-\sin x)\cos(\sqrt{\sin x+\cos x})}{2\sqrt{\sin x+\cos x}}$

24. $e^x[2\cot 2x+\log(\sin 2x)]$ **25.** $\frac{4x}{(1+x^2)^2}.\sin\left(\frac{1-x^2}{1+x^2}\right)^2$ **26.** $\frac{4x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$

27. $2(\sin x+x^2)\sec^2 2x+(\cos x+2x)\tan 2x$