AREA RELATED TO CIRCLE

CONTENTS

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PERIMETER AND AREA OF A CIRCLE

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains same.

The fixed point is called the centre and the given constant distance is known as the radius of the circle.

The perimeter of a circle is known as its circumference.

If r is the radius of a circle, then

(i) Circumference = $2\pi r$ or πd, where $d = 2r$ is the diameter of the circle.

(ii) Area = πr^2 or $\pi d^2/4$

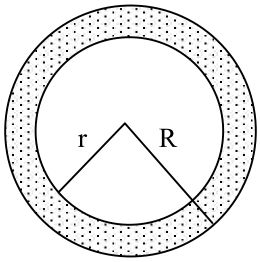
(iii) Area of semi-circle = $\dfrac{\pi r^2}{2}$

(iv) Area of a quadrant of a circle = $\dfrac{\pi r^2}{4}$

AREA ENCLOSED BY TWO CONCENTRIC CIRCLES

If R and r are radii of two con-centric circles, then area enclosed by the two circles

$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = \pi (R + r) (R - r)$



Some useful results :

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in one minute

$= \dfrac{\text{Distance moved in one minute}}{\text{Circumference}}$

❖ EXAMPLES ❖

Ex.1 Find the area of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

Circumference = 22 cm

$\Rightarrow\ 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow$ r = 7/2 cm

$\therefore$ Area of the circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ cm^2

= 38.5 cm^2

Ex.2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

Circumference = 22 cm

$\Rightarrow 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm

$\therefore$ Area of a quadrant = $\frac{1}{4}\pi r^2$

$= \left\{\frac{1}{4} \times \frac{22}{7}\left(\frac{7}{2}\right)^2\right\}$ cm^2

$= \left\{\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\}$ cm^2

$= \frac{77}{4}$ cm^2 = 9.625 cm^2

Ex.3 If the perimeter of a semi-circular protractor is 66 cm, find the diameter of the protractor (Take π = 22/7).

Sol. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

$\Rightarrow$ 1/2(2 πr) = 66

$\left[\because \text{Perimeter of semi-circle} = \frac{1}{2}(2\pi r)\right]$

$\Rightarrow \pi r = 66 \Rightarrow 22/7 \times r = 66 \Rightarrow r = 21$ cm

$\therefore$ Diameter of the protractor = 2r = (2 × 21) cm

= 42 cm

Ex.4 The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then,

Diameter = 2r cm and Circumference = 2πr cm

It is given that the circumference exceeds the diameter by 16.8 cm

$\therefore$ Circumference = Diameter + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2 \times \frac{22}{7} \times r = 2r + 16.8$ $\left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow 44r = 14r + 16.8 \times 7$

$\Rightarrow 44r - 14r = 117.6 \Rightarrow 30r = 117.6$

$\Rightarrow r = \frac{117.6}{30} = 3.92$

Hence, radius = 3.92 cm

Ex.5 Two circles touch externally. The sum of their areas is 130 π sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.

Sol. If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the raddi of the two circles be r_1 cm and r_2 cm respectively.

Let C_1 and C_2 be the centres of the given circles. Then,

$C_1C_2 = r_1 + r_2$

$\Rightarrow 14 = r_1 + r_2$ [$\because C_1C_2 = 14$ cm (given)]

$\Rightarrow r_1 + r_2 = 14$ ....(i)

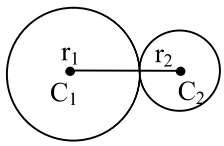
It is given that the sum of the areas of two circles is equal to 130 π cm^2.

$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow r_1^2 + r_2^2 = 130$ ....(ii)

Now,

$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$



$\Rightarrow 14^2 = 130 + 2r_1r_2$ [Using (i) and (ii)]

$\Rightarrow 196 - 130 = 2r_1r_2$

$\Rightarrow \quad r_1 r_2 = 33$ ....(iii)

Now,

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$\Rightarrow \quad (r_1 - r_2)^2 = 130 - 2 \times 33$

[Using (ii) and (iii)]

$\Rightarrow \quad (r_1 - r_2)^2 = 64 \;\Rightarrow\; r_1 - r_2 = 8$ ....(iv)

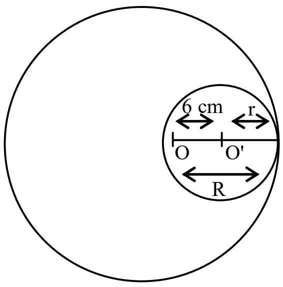
Solving (i) and (iv), we get $r_1 = 11$ cm

and $r_2 = 3$ cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

Ex.6 Two circles touch internally. The sum of their areas is $116\,\pi$ cm^2 and distance between their centres is 6 cm. Find the radii of the circles.

Sol. Let R and r be the radii of the circles having centres at O and O' respectively. Then,



Sum of areas = 116π cm^2

$\Rightarrow \quad \pi R^2 + \pi r^2 = 116\,\pi$

$\Rightarrow \quad R^2 + r^2 = 116$ ....(i)

Distance between the centres = 6 cm

$\Rightarrow \quad OO' = 6$ cm

$\Rightarrow \quad R - r = 6$ ....(ii)

Now, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$\Rightarrow \quad (R + r)^2 + 36 = 2 \times 116$

[Using (i) and (ii)]

$\Rightarrow \quad (R + r)^2 = (2 \times 116 - 36) = 196$

$\Rightarrow \quad R + r = 14$ .....(iii)

Solving (ii) and (iii), we get R = 10 and r = 4.

Hence, radii of the given circles are 10 cm and 4 cm respectively.

Ex.7 A copper wire, when bent in the form of a square, encloses an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it (Use $\pi = 22/7$).

Sol. We have,

Area of the square = 484 cm^2

$\therefore$ Side of the square $\sqrt{484}$ cm= 22 cm

$$\left[\begin{array}{l} \because \text{Area} = (\text{Side})^2 \\ \therefore \text{Side} = \sqrt{\text{Area}} \end{array}\right]$$

So, Perimeter of the square = 4 (side)

= (4 × 22) cm = 88 cm

Let r be the radius of the circle. Then,

Circumference of the circle = Perimeter of the square.

$\Rightarrow \quad 2\pi r = 88$

$\Rightarrow 2 \times 22/7 \times r = 88$

$\Rightarrow r = 14$ cm

$\therefore$ Area of the circle = πr^2

$= \left\{\frac{22}{7} \times (14)^2\right\}$ cm^2

= 616 cm^2

Ex.8 A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.

Sol. We have,

Length of the wire = Circumference of the circle

Length of the wire $= \left\{2 \times \frac{22}{7} \times 28\right\}$ cm

[Using $C = 2\pi r$]

Length of the wire = 176 cm ....(i)

Let the side of the square be x cm. Then,

Perimeter of the square = Length of the wire

$\Rightarrow \quad 4x = 176$ [Using (i)]

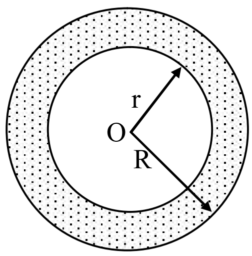
$\Rightarrow \quad x = 44$ cm

Hence, the length of the sides of the square is 44 cm.

Ex.9 A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

Sol. Let the outer and inner radii of the ring

be R metres and r metres respectively. Then,



$2\pi R = 396$ and $2\pi r = 352$

$\Rightarrow$ $2 \times 22/7 \times R = 396$ and $2 \times 22/7 \times r = 352$

$\Rightarrow$ $R = 396 \times 7/22 \times 1/2$ and $r = 352 \times 7/22 \times 1/2$

$\Rightarrow$ $R = 63$ m and $r = 56$ m

Hence, width of the track = (R – r) metres

= (63 – 56) metres = 7 metres

Ex.10 The inner circumference of a circular track is 220 m. The track is 7m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹ 2 per metre. (Use $\pi = 22/7$)

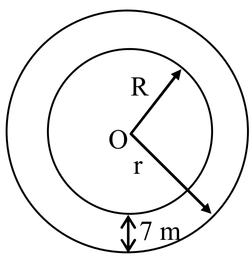
Sol. Let the inner and outer radii of the circular track be r metres and R metres respectively. Then,

Inner circumference = 220 metres

$\Rightarrow$ $2\pi r = 220 \Rightarrow 2 \times 22/7 \times r = 220 \Rightarrow r = 35$ m

Since the track is 7 metre wide everywhere. Therefore,

R = Outer radius = r + 7 = (35 + 7) m = 42 m



$\therefore$ Outer circumference

$= 2\pi R = 2 \times 22/7 \times 42$ m = 264 m

Rate of fencing = ₹ 2 per metre

$\therefore$ Total cost of fencing

= (Circumference × Rate) = ₹ (264 × 2) = ₹ 528

Ex.11 A bicycle whell makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Sol. Let the radius of the wheel be r cm.

Distance covered by the wheel in one revolution

$$= \frac{\text{Distance moved}}{\text{Number of revolutions}} = \frac{11}{5000}\text{km}$$

$$= \frac{11}{5000} \times 1000 \times 100 \text{ cm} = 220 \text{ cm}$$

$\therefore$ Circumference of the wheel = 220 cm

$\Rightarrow$ $2\pi r = 220$ cm $\Rightarrow$ $2 \times 22/7 \times r = 220$

$\Rightarrow$ r = 35 cm

$\therefore$ Diameter = 2r cm = (2× 35) cm = 70 cm

Hence, the diameter of the wheel is 70 cm.

Ex.12 A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour ?

Sol. We have,

Speed of the car = 66 km/hr

$\therefore$ Distance travelled by the car in 1 hour = 66 km

$\Rightarrow$ Distance travelled by the car in 10 min.

$$= \frac{66}{60} \times 10 \text{ km} = 11 \text{ km} = 11 \times 1000 \times 100 \text{ cm}$$

We have,

Radius of car wheels = 40 cm

$\therefore$ Circumference of the wheels

$= 2 \times 22/7 \times 40$ cm

$\Rightarrow$ Distance travelled by the car when its wheels take one complete revolution

$= 2 \times 22/7 \times 40$ cm

$\therefore$ Number of revolutions made by the wheels in 10 minutes

$$= \frac{\text{Distance covered by the car in 10 minutes}}{\text{Distance covered by the car when its wheels make one complete revolution}}$$

$$= \frac{11 \times 100 \times 100}{2 \times \frac{22}{7} \times 40} = \frac{11 \times 1000 \times 100 \times 7}{2 \times 22 \times 40} = 4375$$

Hence, each wheel makes 4375 revolutions in 10 minutes.

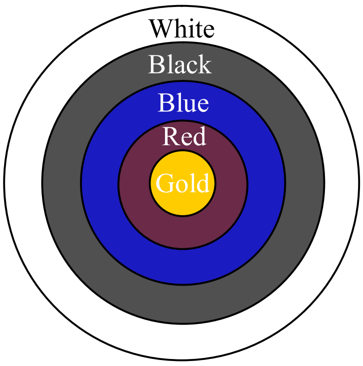
Ex.13 Fig. depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Sol. We have,

r = Radius of the region representing Gold score = 10.5 cm

$\therefore$ r_1 = Radius of the region representing Gold and Red scoring areas

= (10.5 + 10.5) cm = 21 cm = 2r cm



r_2 = Radius of the region representing Gold, Red and Blue scoring areas

= (21 + 10.5) cm = 31.5 cm = 3r cm

r_3 = Radius of the region representing Gold, Red, Blue and Black scoring areas

= (31.5 + 10.5) cm = 42 cm = 4r cm

r_4 = Radius of the region representing Gold, Red, Blue, Black and white scoring areas

= (42 + 10.5) cm = 52.5 cm = 5r cm

Now, A_1 = Area of the region representing Gold scoring area

$= \pi r^2 = 22/7 \times (10.5)^2 = 22/7 \times 10.5 \times 10.5$

$= 22 \times 1.5 \times 10.5 = 346.5 \text{ cm}^2$

A_2 = Area of the region representing Red scoring area

$= \pi(2r)^2 - \pi r^2 = 3\pi r^2 = 3A_1 = 3 \times 346.5 \text{ cm}^2$

$= 1039.5 \text{ cm}^2$

A_3 = Area of the region representing Blue scoring area

$= \pi (3r)^2 - \pi(2r)^2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2 = 5A_1$

$= 5 \times 346.5 \text{ cm}^2 = 1732.5 \text{ cm}^2$

A_4 = Area of the region representing Black scoring area

$= \pi (4r)^2 - \pi(3r)^2 = 7\pi r^2$

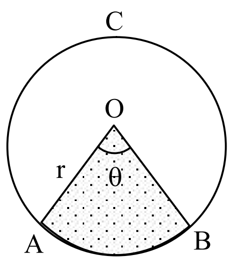
$= 7 A_1 = 7 \times 346.5 \text{ cm}^2 = 2425.5 \text{ cm}^2$

A_5 = Area of the region representing White scoring area

$= \pi(5r)^2 - \pi(4r)^2 = 9\pi r^2$

$= 9 A_1 = 9 \times 346.5 \text{ cm}^2 = 3118.5 \text{ cm}^2$

SECTOR OF A CIRCLE



Minor sector :

A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary

In Fig. sector OAB is the minor sector.

Major sector :

A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary.

In Fig. sector OACB is the major sector.

Following are some important points to remember:

(i) A minor sector has an angle θ, subtended at the centre of the circle, whereas a major sector has no angle.

(ii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

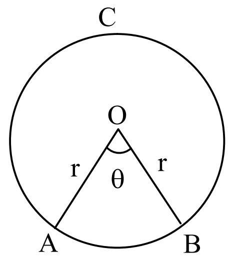
(iii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iv) The boundary of a sector consists of an arc of the circle and the two radii.

➤ AREA OF A SECTOR

If the arc subtends an angle of θ at the centre, then its arc length is

$$\frac{\theta}{180} \times \pi r$$



Hence, the arc length ℓ of a sector of angle θ in a circle of radius r is given by

$$\ell = \frac{\theta}{180} \times \pi r \qquad \text{....(i)}$$

If the arc subtends an angle θ, then area of the corresponding sector is

$$\frac{\pi r^2 \theta}{360}$$

Thus, the area A of a sector of angle θ in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{\theta}{360} \times \text{(Area of the circle)} \qquad \text{....(ii)}$$

Some useful results to remember:

(i) Angle described by minute hand in 60 minutes = 360º

∴ Angle described by minute hand in one minute

$$= \left(\frac{360}{60}\right)^{\circ} = 6^{\circ}$$

Thus, minute hand rotates through an angle of 6º in one minute.

(ii) Angle described by hour hand in 12 hours = 360º

∴ Angle described by hour hand in one hour

$$= \left(\frac{360}{12}\right)^{\circ}$$

$$= 30^{\circ}$$

❖ EXAMPLES ❖

Ex.14 A sector is cut from a circle of radius 21 cm. The angle of the sector is 150º.Find the length of its arc and area.

Sol. The arc length l and area A of a sector of angle θ in a circle of radius r are given by

$$l = \frac{\theta}{360} \times 2\pi r \text{ and } A = \frac{\theta}{360} \times \pi r^2$$

respectively.

Here, r = 21 cm and θ = 150

$$\therefore l = \left\{\frac{150}{360}\times 2\times\frac{22}{7}\times 21\right\} \text{cm} = 55 \text{ cm}$$

$$\text{and } A = \left\{\frac{150}{360}\times\frac{22}{7}\times(21)^2\right\} \text{cm}^2 = \frac{1155}{2} \text{cm}^2$$

$$= 577.5 \text{ cm}^2$$

Ex.15 Find the area of the sector of a circle whose radius is 14 cm and angle of sector is 45º.

Sol. We know that the area A of a sector of angle θ in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

Here, r = 14 cm and θ = 45

$$\therefore A = \left\{\frac{45}{360}\times\frac{22}{7}\times(14)^2\right\} \text{cm}^2$$

$$= \left\{\frac{1}{8}\times\frac{22}{7}\times14\times14\right\} \text{cm}^2$$

$= 77\ \text{cm}^2$

Ex.16 In Fig. there are shown sectors of two concentric circles of radii 7 cm and 3.5 cm. Find the area of the shaded region.

(Use $\pi = 22/7$).

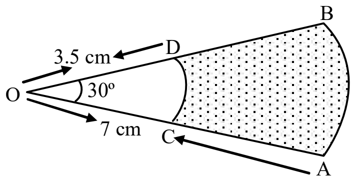
Sol. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30º in a circle of radius 7 cm

$$\Rightarrow A_1 = \left\{\frac{30}{360}\times\frac{22}{7}\times7^2\right\} \text{cm}^2$$

$$\left[\text{Using: } A = \frac{\theta}{360}\times\pi r^2\right]$$

$\Rightarrow A_1 = 77/6\ \text{cm}^2$

A_2 = Area of a sector of angle 30º in a circle of radius 3.5 cm.



$$\Rightarrow A_2 = \left\{\frac{30}{360}\times\frac{22}{7}\times(3.5)^2\right\} \text{cm}^2$$

$$\Rightarrow A_2 = \left\{\frac{1}{12}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right\} \text{cm}^2 = \frac{77}{24}\ \text{cm}^2$$

$\therefore$ Area of the shaded region

$$= A_1 - A_2 = \left(\frac{77}{6}-\frac{77}{24}\right)\text{cm}^2$$

$$= \frac{77}{24}\times(4-1)\ \text{cm}^2 = 77/8\ \text{cm}^2 = 9.625\ \text{cm}^2$$

Ex.17 A pendulum swings through an angle of 30º and describes an arc 8.8 cm in length. Find the length of the pendulum.

Sol. Here, θ = 30º, l = arc = 8.8 cm

$$\therefore l = \frac{\theta}{360}\times 2\pi r \Rightarrow 8.8 = \frac{30}{360}\times 2\times 22/7\times r$$

$$\Rightarrow r = \frac{8.8\times6\times7}{22}\ \text{cm} = 16.8\ \text{cm}$$

Ex.18 The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use $\pi = 22/7$)

Sol. Clearly, minute hand of a clock describes a circle of radius equal to its length i.e., 14 cm.

Since the minute hand rotates through 6º in one minute. Therefore, area swept by the minute hand in one minute is the area of a sector of angle 6º in a circle of radius 14 cm. Hence, required area A is given by

$$A = \frac{\theta}{360}\times\pi r^2$$

$$\Rightarrow A = \left\{\frac{6}{360}\times\frac{22}{7}\times(14)^2\right\} \text{cm}^2$$

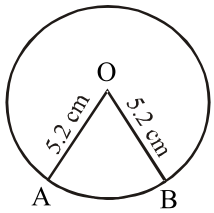
$$\Rightarrow A = \left\{\frac{1}{60}\times\frac{22}{7}\times14\times14\right\} \text{cm}^2 = \frac{154}{15}\ \text{cm}^2$$

$= 10.26\ \text{cm}^2$

Ex.19 The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Sol. Let OAB be the given sector. Then,

Perimeter of sector OAB = 16.4 cm



$\Rightarrow$ OA + OB + arc AB = 16.4 cm

$\Rightarrow$ 5.2 + 5.2 + arc AB = 16.4

$\Rightarrow$ arc AB = 6 cm $\Rightarrow l = 6$ cm

$\therefore$ Area of sector OAB = $\frac{1}{2} lr$

$$= \frac{1}{2}\times 6\times 5.2\ \text{cm}^2 = 15.6\ \text{cm}^2$$

Ex.20 The minute hand of a clock is 10cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.

Sol. We have,

Angle described by the minute hand in one minute = 6º

$\therefore$ Angle described by the minute hand in 35 minutes = (6 × 35)º = 210º

$\therefore$ Area swept by the minute hand in 35 minutes

= Area of a sector of angle 210º in a circle of radius 10 cm

$= \left\{\frac{210}{360} \times \frac{22}{7} \times (10)^2\right\}$ cm² = 183.3 cm²

$\left[Using\ : A = \frac{\theta}{360^\circ} \times \pi r^2\right]$

Ex.21 The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days.

(Take π = 22/7)

Sol. In 2 days, the short hand will complete 4 rounds.

$\therefore$ Distance moved by its tip = 4 (Circumference of a circle of radius 4 cm)

$= 4 \times \left(2 \times \frac{22}{7} \times 4\right)$ cm = $\frac{704}{7}$ cm

In 2 days, the long hand will complete 48 rounds.

$\therefore$ Distance moved by its tip

= 48 (Circumference of a circle of radius 6 cm)

$= 48 \times \left(2 \times \frac{22}{7} \times 6\right)$ cm = $\frac{12672}{7}$ cm

Hence,

Sum of the distance moved by the tips of two hands of the clock

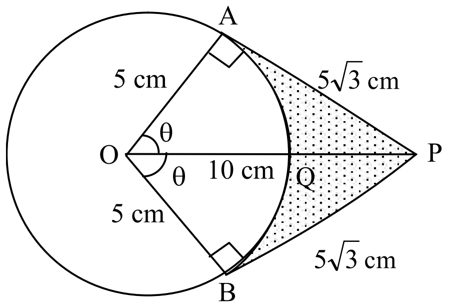
$= \left(\frac{704}{7} + \frac{12672}{7}\right)$ cm = 1910.57 cm

Ex.22 An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.

Sol. In the adjacent figure, let $\angle AOP = \angle BOP = \theta$. Clearly, portion AB of the belt is not in contact with the rim of the pulley. In right triangle OAP, we have

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 60^\circ$

$\Rightarrow \angle AOB = 2\theta = 120^\circ$



$\therefore$ Arc AB $= \frac{120^\circ \times 2 \times \pi \times 5}{360}$ cm = $\frac{10\pi}{3}$ cm

[Using $\ell = \frac{\theta}{360} \times 2\pi r$]

Hence, Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim – Length of arc AB

$= 2\pi \times 5$ cm $- \frac{10\pi}{3}$ cm = $\frac{20\pi}{3}$ cm

Now,

Area of sector OAQB = $\frac{120}{360} \times \pi \times 5^2$ cm²

$= \frac{25\pi}{3}$ cm² $\left[Using\ : Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2 (Area of ΔOAP)

= 2 × (1/2 × OA × AP)

$= 5 \times 5\sqrt{3}$ cm²

$[\because OP^2 = OA^2 + AP^2 \Rightarrow AP = \sqrt{100-25} = 5\sqrt{3}\ ]$

$= 25\sqrt{3}$ cm²

Hence,

Shaded area = Area of quadrilateral

OAPB – Area of sector OAQB.

$$= \left(25\sqrt{3} - \frac{25\pi}{3}\right) \text{cm}^2 = \frac{25}{3}(3\sqrt{3} - \pi) \text{ cm}^2$$

Ex.23 An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. Find the radius of the circle.

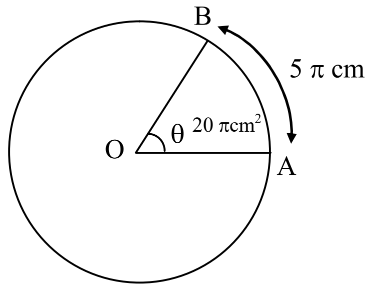
Sol. Let the radius of the circle be r cm and the arc AB of length 5π cm subtends angle θ at the centre O of the circle. Then,

Arc AB = 5π cm and

Area of sector OAB = 20π cm^2

$$\Rightarrow \frac{\theta}{360} \times 2\pi r = 5\pi \text{ and } \frac{\theta}{360} \times \pi r^2 = 20\pi$$

$$\Rightarrow \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r} = \frac{20\pi}{5\pi}$$



$\Rightarrow r/2 = 4 \quad \Rightarrow \quad r = 8$ cm

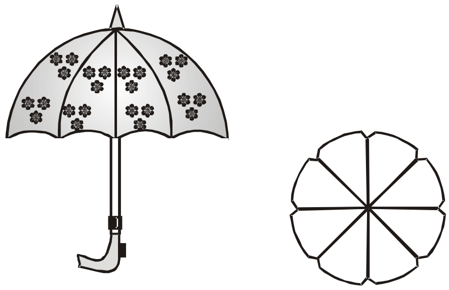
ALTER : We have, Area = $\frac{1}{2} lr \Rightarrow 20\pi$

$$= \frac{1}{2} \times 5\pi \times r = 8 \text{ cm}$$

Ex.24 An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm. Find the area between the two consecutive ribs of the umbrella.

Sol. Since ribs are equally spaced. Therefore,

Angle made by two consecutive ribs at the centre = $\frac{360^\circ}{8} = 45^\circ$



Thus,

Area between two consecutive ribs

= Area of a sector of a circle of radius 45 cm and sector angle 45°

$$= \left\{\frac{45}{360} \times \frac{22}{7} \times 45 \times 45\right\} \text{cm}^2$$

$$\left[\text{Using : Area} = \frac{\theta}{360} \times \pi r^2\right]$$

$$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2 = 795.53 \text{ cm}^2$$

Ex.25 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find:

(i) the total length of the silver wire required

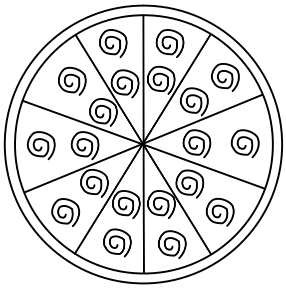
(ii) the area of each sector of the brooch.

Sol. (i) We have,

Total length of the silver wire

= Circumference of the circle of radius

35/2 mm + Length of five diameters



$$= 2\pi \times \frac{35}{2} + 5 \times 35 \text{ mm}$$

$$= \left(2 \times \frac{22}{7} \times \frac{35}{2} + 175\right) \text{ mm}$$

= 285 mm

(ii) The circle is divided into 10 equal sectors, Therefore, Area of each sector of the brooch

= 1/10 (Area of the circle)

= $1/10 \times \pi \times (35/2)^2$ cm^2

$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$ mm^2

$= \frac{385}{4}$ mm^2

➤ SEGMENT OF A CIRCLE & ITS AREA

◈ **Segment of a circle :**

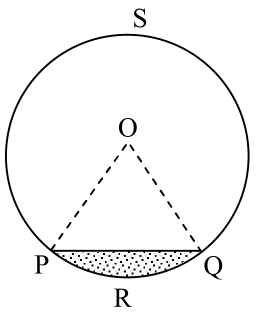
The region enclosed by an arc and a chord is called the segment of the circle.

◈ **Minor segment :**

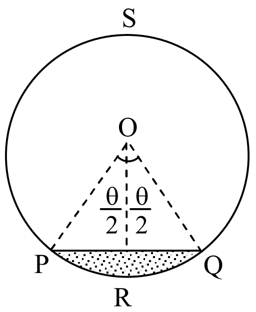
If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment.

◈ **Major segment :**

A segment corresponding a major arc of a circle is known as the major segment.



Area of the sector OPRQ = Area of the segment PRQ + Area of ΔOPQ



$\Rightarrow$ Area of segment PRQ = $\left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$

Ex.26 Find the area of the segment of a circle,given that the angle of the sector is 120º and the radius of the circle is 21 cm. (Take π = 22/7)

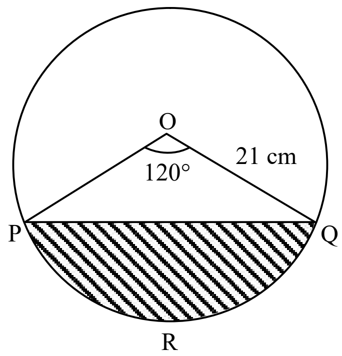
Sol. Here, r = 21 cm and π = 120

$\therefore$ Area of the segment

$$= \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

$$= \left\{\frac{22}{7} \times \frac{120}{360} - \sin 60^\circ \cos 60^\circ\right\} (21)^2 \text{ cm}^2$$

$$= \left\{\frac{22}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right\} (21)^2 \text{ cm}^2$$

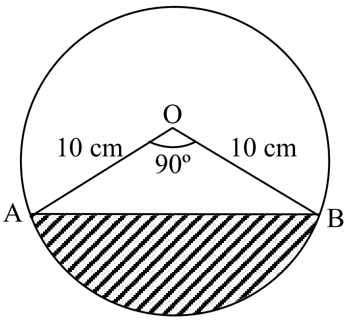


$$= \left\{\frac{22}{21} \times (21)^2 - (21)^2 \times \frac{\sqrt{3}}{4}\right\} \text{cm}^2$$

$$= \left(462 - \frac{441}{4}\sqrt{3}\right) \text{cm}^2 = \frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^2$$

Ex.27 A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of major and minor segments (Take π = 3.14)

Sol. We know that the area of a minor segment of angle θ° in a circle of radius r is given by



$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

Here, r = 10 and θ = 90°

$$\therefore \quad A = \left\{\frac{3.14 \times 90}{4} - \sin 45^\circ \cos 45^\circ\right\} (10)^2 \text{ cm}^2$$

$$\Rightarrow \quad A = \left\{\frac{3.14}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right\} 10)^2 \text{ cm}^2$$

$$\Rightarrow \quad A = \{3.14 \times 25 - 50\}\text{cm}^2 = (78.5 - 50) \text{ cm}^2$$

$= 28.5 \text{ cm}^2$

Area of the major segment = Area of the circle – Area of the minor segment

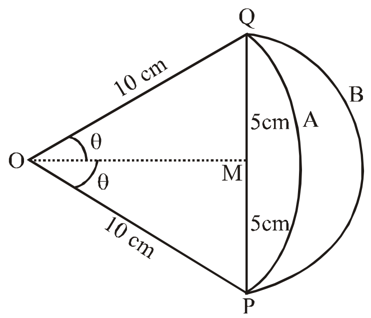
$= \{3.14 \times 10^2 - 28.5\} \text{ cm}^2$

$= (314 - 28.5) \text{ cm}^2 = 285.5 \text{ cm}^2$

Ex.28 The diagram shows two arcs, A and B. Arc A is part of the circle with centre O and radius of PQ. Arc B is part of the circle with centre M and radius PM, where M is the mid-point of PQ. Show that the area enclosed by the two arcs is equal to $25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2$.

Sol. We have,

Area enclosed by arc B and chord PQ = Area of semi-circle of radius 5 cm



$$= 1/2 \times \pi \times 5^2 \text{ cm}^2 = \frac{25\pi}{2} \text{ cm}^2$$

Let ∠MOQ = ∠MOP = θ

In ΔOMP, we have

$$\sin\theta = \frac{PM}{OP} = \frac{5}{10} = \frac{1}{2}$$

$\Rightarrow \theta = 30^\circ \Rightarrow \angle POQ = 2\theta = 60^\circ$

∴ Area enclosed by arc A and chord PQ.

= Area of segment of circle of radius 10 cm and sector containing angle 60°

$$= \left\{\frac{\pi \times 60}{360} - \sin 30^\circ \times \cos 30^\circ\right\} \times 10^2 \text{ cm}^2$$

$$\left[\because A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$$

$$= \left\{\frac{50\pi}{3} - 25\sqrt{3}\right\} \text{cm}^2$$

Hence,

$$\text{Required area} = \left\{\frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)\right\} \text{cm}^2$$

$$\Rightarrow \text{Required area} = \left\{25\sqrt{3} - \frac{25\pi}{6}\right\} \text{ cm}^2$$

$$= 25\left\{\sqrt{3} - \frac{\pi}{6}\right\} \text{ cm}^2$$

➤ AREAS OF COMBINATIONS OF PLANE FIGURES

Ex.29 The inner and outer diameters of ring I of a dartboard are 32 cm and 34 cm respectively and those of rings II are 19cm and 21 cm respectively. What is the total area of these two rings ?

Sol. We have,

Area of ring I = $(\pi \times 17^2 - \pi \times 16^2) \text{ cm}^2$

$$= \frac{22}{7} \times (17^2 - 16^2) \text{ cm}^2$$

$$= \frac{22}{7} \times (17 + 16)(17 - 16) \text{ cm}^2$$

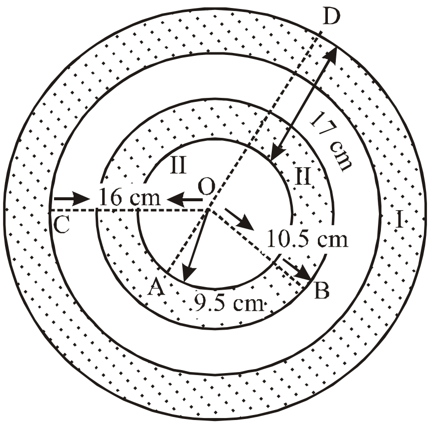
$= \frac{22}{7} \times 33\ cm^2$

Area of ring II = $(\pi \times 10.5^2 - \pi \times 9.5^2)\ cm^2$

$= \pi\ (10.5^2 - 9.5^2)\ cm^2$

$= \frac{22}{7} \times (10.5 + 9.5)\ (10.5 - 9.5)\ cm^2$

$= \frac{22}{7} \times 20\ cm^2$



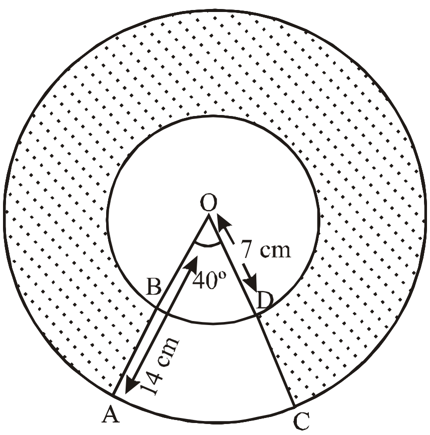
Hence,

Total area of two rings = $\frac{22}{7} \times 33 + \frac{22}{7} \times 20\ cm^2$

$= \frac{22}{7} \times (33 + 20)\ cm^2 = 166.57\ cm^2$

Ex.30 Find the area of the shaded region in Fig. where radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

Sol. We have,



Area of the region ABDC = Area of sector AOC – Area of sector BOD

$= \left(\frac{40}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{40}{360} \times \frac{22}{7} \times 7 \times 7\right)\ cm^2$

$= \left(\frac{1}{9} \times 22 \times 14 \times 2 - \frac{1}{9} \times 22 \times 7 \times 1\right)\ cm^2$

$= \frac{22}{9} = \frac{154}{3} \times (28 - 7)\ cm^2 = cm^2$

Area of the circular ring

$= \left(\frac{22}{7} \times 14 \times 14 - \frac{22}{7} \times 7 \times 7\right) cm^2$

$= (22 \times 14 \times 2 - 22 \times 7 \times 1) cm^2$

$= 22 \times 21\ cm^2 = 462\ cm^2$

Hence,

Required shaded area = $\left(462 - \frac{154}{3}\right)\ cm^2$

$= \frac{1232}{3}\ cm^2 = 410.67\ cm^2$

Ex.31 AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30^\circ$, find the area of the shaded region.

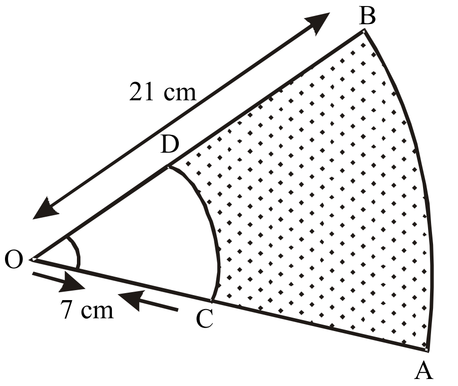
Sol. We have,

Shaded area = Area of sector OAB – Area of sector OCD

⇒ Shaded area

$= \left(\frac{30}{360} \times \frac{22}{7} \times 21 \times 21 - \frac{30}{360} \times \frac{22}{7} \times 7 \times 7\right)\ cm^2$

$= \frac{30}{360} \times \frac{22}{7} \times (21 \times 21 - 7 \times 7)\ cm^2$



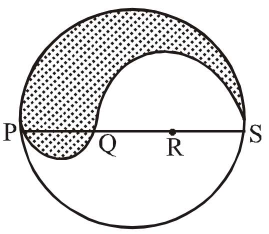
$$= \frac{11}{42} \times (21 + 7) \times (21 - 7)\ \text{cm}^2$$

$$= \frac{11}{42} \times 28 \times 14\ \text{cm}^2 = 102.67\ \text{cm}^2$$

Ex.32 PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region.

Sol. We have,

PS = Diameter of a circle of radius 6 cm = 12 cm



$\therefore$ PQ = QR = RS = 12/3 = 4 cm

QS = QR + RS = (4 + 4) cm = 8 cm

Hence, required perimeter

= Arc of semi-circle of radius 6 cm

+ Arc of semi-circle of radius 4 cm

+ Arc of semi-circle of radius 2 cm

$= (\pi \times 6 + \pi \times 4 + \pi \times 2)$ cm $= 12\pi$ cm

Required area = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter – Area of semi-circle with QS as diameter.

$$= \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$$

$$= \frac{1}{2} \times \frac{22}{7}\ (6^2 + 2^2 - 4^2)$$

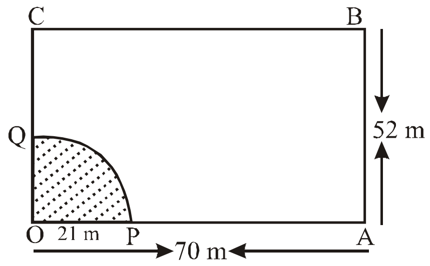
$$= \frac{1}{2} \times \frac{22}{7} \times 24$$

$$= \frac{264}{7} = 37.71\ \text{cm}^2$$

Ex.33 A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze ?

Sol. Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius r = 21 m.

$\therefore$ Required area $= \frac{1}{4}\pi r^2$



$\Rightarrow$ Required area $= \left\{\frac{1}{4} \times \frac{22}{7} \times (21)^2\right\}$ cm^2

$$= \frac{693}{2}\ \text{cm}^2 = 346.5\ \text{cm}^2$$

Ex.34 A paper is in the form of a rectangle ABCD in which AB = 20 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of a remaining part.

Sol. We have,

Length of the rectangle ABCD = AB = 20 cm

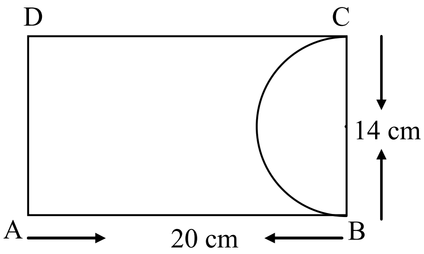
Breadth of the rectangle ABCD = BC = 14 cm

$\therefore$ Area of rectangle ABCD = (20 × 14) cm^2

= 280 cm^2

Diameter of the semi-circle = BC = 14 cm

$\therefore$ Radius of the semi-circle = 7 cm



Area of the semi-circular portion cut off from the rectangle ABCD

$= 1/2\ (\pi r^2) = \left(\frac{1}{2} \times \frac{22}{7} \times 7^2\right) cm^2 = 77\ cm^2$

$\therefore$ Area of the remaining part

= Area of rectangle ABCD – Area of semi-circle

$= (280 - 77)\ cm^2 = 203\ cm^2$

Ex.35 In figure, find the area of the shaded region

$[\text{Use } \pi = 3.14]$

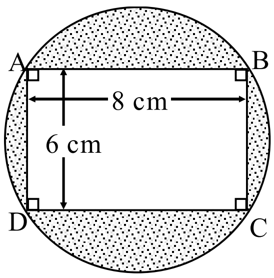
Sol. Clearly, Diameter of the circle = Diagonal BD of rectangle ABCD

$\therefore$ Diameter $= BD = \sqrt{BC^2 + CD^2}$

$= \sqrt{6^2 + 8^2}\ cm = 10\ cm$

Let r be the radius of the circle. Then,

$r = (10/2)\ cm = 5\ cm$



Area of rectangle ABCD = AB × BC

$= (8 \times 6)\ cm^2 = 48\ cm^2$

Area of the circle $= \pi r^2 = 3.14 \times (5)^2\ cm^2$

$= 78.50\ cm^2$

Hence,

Area of the shaded region = Area of the circle – Area of rectangle ABCD

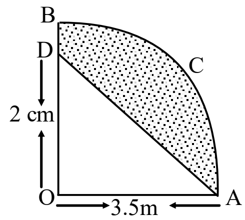
$= (78.50 - 48)\ cm^2 = 30.50\ cm^2$

Ex.36 In Fig. AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion

(Take $\pi = 22/7$).

Sol. We have,

Area of quadrant AOBCA



$= \frac{1}{4}\pi r^2 = 1/4 \times 22/7 \times (3.5)^2$

$= 1/4 \times 22/7 \times 7/2 \times 7/2$

$= 27/8\ cm^2 = 9.625\ cm^2$

Area of $\Delta AOD = 1/2 \times$ Base × Height

$= 1/2\ (OA \times OB) = 1/2\ [3.5 \times 2]\ cm^2 = 3.5\ cm^2$

Hence,

Area of the shaded portion

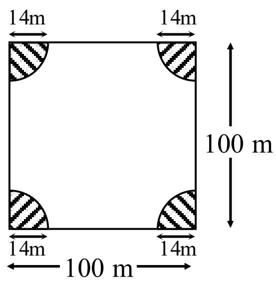
= Area of quadrant – Area of ΔAOD

$= (9.625 - 3.5)\ cm^2 = 6.125\ cm^2$

Ex.37 A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. Find the area of the remaining part of the park (Use $\pi = 22/7$).

Sol. We have,

Area of each quadrant of a circle of radius 14 m



$= 1/4\ (\pi r^2) = 1/4 \times 22/7 \times 14 \times 14\ [\because r = 14]$

$= 154\ m^2$

$\therefore$ Area of 4 quadrants $= (4 \times 154)\ m^2 = 616\ m^2$

Area of square park having side 100 m long

$= (100 \times 100)\ m^2 = 10{,}000\ m^2$

Hence,

Area of the remaining part of the park

$= 10{,}000 - 616 = 9384\ m^2$

Ex.38 ABCD is a flower bed. If OA = 21 cm and OC = 14 m, find the area of the bed.

[Take $\pi = 22/7$]

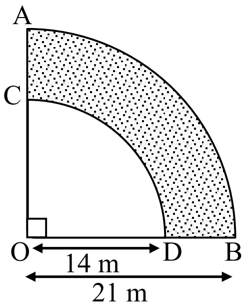
Sol. We have,

OA = R = 21 m and OC = r = 14 m

$\therefore$ Area of the flower bed

= Area of a quadrant of a circle of radius R

– Area of a quadrant of a circle of radius r



$$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2 = \frac{\pi}{4}(R^2 - r^2)$$

$$= \frac{1}{4} \times \frac{22}{7}(21^2 - 14^2)\ cm^2$$

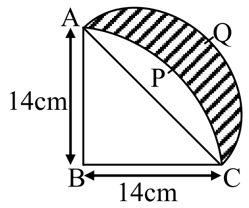
$[\because R = 21 \text{ m and } r = 14 \text{ m}]$

$$= \left\{\frac{1}{4} \times \frac{22}{7} \times (21+14)\ (21-14)\right\}\ m^2$$

$$= \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\} m^2 = 192.5\ m^2$$

Ex.39 ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded portion.

Sol. In the right-angled triangle ABC, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 14^2 + 14^2 \Rightarrow AC = \sqrt{2 \times 14^2} = 14\sqrt{2}$ cm

$\Rightarrow \dfrac{AC}{2} = \dfrac{14\sqrt{2}}{2}$ cm $= 7\sqrt{2}$ cm

Now,

Required area

= Area APCQA

= Area ACQA – Area ACPA

= Area ACQA – (Area ABCPA – Area of ΔABC)

= (Area of semi-circle with AC as diameter)

– [Area of a quadrant of a circle with AB as radius – Area of ΔABC]

$$= \left[\frac{1}{2}\left\{\frac{22}{7} \times (7\sqrt{2})^2\right\} - \left\{\frac{1}{4} \times \frac{22}{7} \times 14^2 - \frac{1}{2} \times 14 \times 14\right\}\right]$$

$$= \left\{\frac{1}{2} \times \frac{22}{7} \times 49 \times 2 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 + \frac{1}{2} \times 14 \times 14\right\} cm^2$$

$= (154 - 154 + 98)\ cm^2 = 98\ cm^2$

Ex.40 It is proposed to add to a square lawn measuring 58 cm on a side, two circular ends. The centre of each circle being the point of intersection of the diagonals of the square. Find the area of the whole lawn.

Sol. We have,

Length of the diagonal of the square

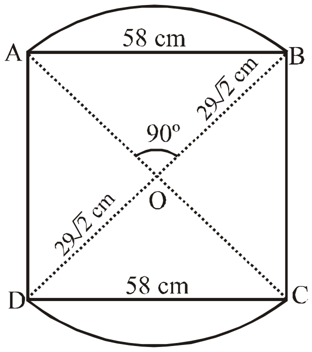
$$= \sqrt{58^2 + 58^2} = 58\sqrt{2}\ \text{cm}$$

So, radius of the circle having centre at the point of intersection of diagonals is $29\sqrt{2}$ cm.

Now,

Area of one circular end

= Area of a segment of angle 90º in a circle of radius $29\sqrt{2}$ cm



$$= \left\{\frac{22}{7} \times \frac{90}{360} - \sin 45^{\circ} \cos 45^{\circ}\right\} \times (29\sqrt{2})^2 \text{ cm}^2$$

$$\left[\because \text{Area} = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$$

$$= \left(\frac{11}{14} - \frac{1}{2}\right) \times 29 \times 29 \times 2 \text{ cm}^2$$

$$= 29 \times 29 \times 2 \times \frac{4}{14} \text{ cm}^2 = \frac{3364}{7} \text{ cm}^2$$

$\therefore$ Area of the whole lawn

= Area of the square + 2 (Area of a circular end)

$$= \left\{58 \times 58 + 2 \times \frac{3364}{7}\right\} \text{cm}^2$$

$$= \left\{3364 + 2 \times \frac{3364}{7}\right\} \text{cm}^2$$

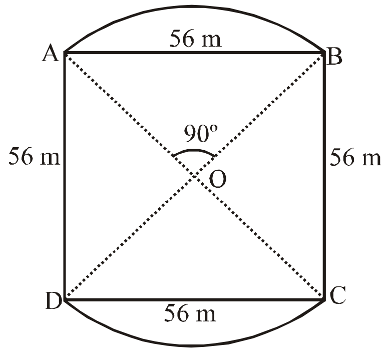
$$= 3364\left(1 + \frac{2}{7}\right) \text{ cm}^2 = 3364 \times 9/7 \text{ cm}^2$$

$= 4325.14 \text{ cm}^2$

Ex.41 In Fig. two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 cm. If the centre of each circular flower bed is the point of intersection of the diagonals of the square lawn, find the sum of the areas of the lawns and the flower beds.

Sol. We have,

$$AC = BD = \sqrt{56^2 + 56^2} = 56\sqrt{2} \text{ m}$$



$\therefore$ OA = OB = 1/2 AC = $58\sqrt{2}$ m

So, radius of the circle having centre at the point of intersection of diagonals is $28\sqrt{2}$ cm.

Now,

Area of one circular end = Area of a segment of angle 90º in a circle of radius $28\sqrt{2}$ m.

$$= \left\{\frac{22}{7} \times \frac{90}{360} - \sin 45^{\circ} \cos 45^{\circ}\right\} \times (28\sqrt{2})^2 \text{ cm}^2$$

$$\left[\begin{array}{l}\text{Substituting } r = 28\sqrt{2} \text{ and } \theta = 90^{\circ} \\ \text{in Area} = \left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) r^2\end{array}\right]$$

$$= \left\{\frac{11}{14} - \frac{1}{2}\right\} \times 28 \times 28 \times 2 \text{ cm}^2$$

$= 28 \times 28 \times 2 \times 4/14 \text{ cm}^2$

$= 448 \text{ cm}^2$

$\therefore$ Area of two flower beds

$= 2 \times 448 \text{ m}^2 = 896 \text{ m}^2$

Area of the square lawn

$= 56 \times 56 \text{ m}^2 = 3136 \text{ m}^2$

Hence,

Total area = $(3136 + 896) \text{ m}^2$

$= 4032 \text{ m}^2$

Ex.42 Find the area of the shaded region in Fig. where ABCD is a square side 10 cm. (Use $\pi = 3.14$)

Sol. Let us mark the four unshaded regions as R_1, R_2, R_3 and R_4.

We have,

Area of R_1 + Area of R_3

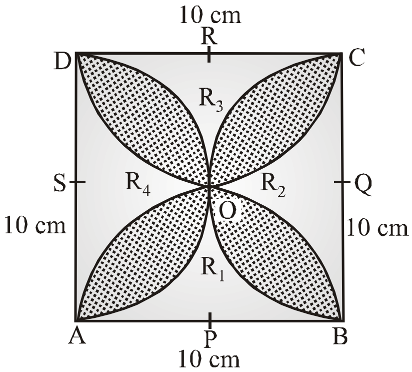
= Area of square ABCD – Area of two semi-circles having centres at Q and S

$$= \left(10\times10-2\times\frac{1}{2}\times3.14\times5^2\right)\text{cm}^2$$

[$\because$ Radius = AP = 5 cm]

= (100 – 3.14 × 25) cm^2 = (100 – 78.5) cm^2

= 21.5 cm^2



Similarly, we have

Area of R_2 + Area of R_4 = 21.5 cm^2

$\therefore$ Area of the shaded region

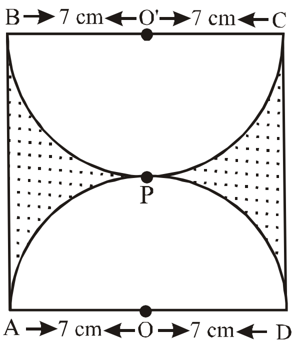
= Area of square ABCD –

(Area of R_1 + Area of R_2 + Area of R_3 + Area of R_4)

= (100 – 2 × 21.5) cm^2 = 57 cm^2

Ex.43 Find the area of the shaded region in Fig. If ABCD is a square of side 14 cm and APD and BPC are semi-circles.

Sol. We have,



Area of the shaded region

= Area of square ABCD – Area of two semi-circles

$$= 14 \times 14 \text{ cm}^2 - 2\left(\frac{1}{2}\times\frac{22}{7}\times7^2\right)\text{ cm}^2$$

= 196 cm^2 – 154 cm^2 = 42 cm^2

Ex.44 ABCD is a field in the shape of a trapezium. AB || DC and $\angle$ABC = 90º, $\angle$DAB=60º. Four sectors are formed with centers A, B, C and D. The radius of each sector is 17.5 m. Find the

(i) total area of the four sectors.

(ii) area of remaining portion given that AB = 75m and CD = 50 m.

Sol. Since AB || CD and $\angle$ABC = 90º. Therefore

$\angle$BCD = 90º. Also, $\angle$BAD = 60º

$\therefore$ $\angle$CDA = 180º – 60º = 120º

[Co-interior angles]

(i) We have,

Total area of the four sectors

= Area of sector at A + Area of sector at B

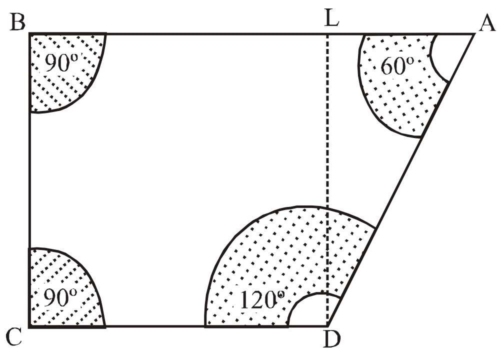
+ Area of sector at C + Area of sector at D.

$$= \frac{60}{360}\times\pi\times(17.5)^2+\frac{90}{360}\times\pi\times(17.5)^2$$

$$+\frac{90}{360}\times\pi\times(17.5)^2+\frac{120}{360}\times\pi\times(17.5)^2$$

$$= \left\{\left(\frac{1}{6}+\frac{1}{4}+\frac{1}{4}+\frac{1}{3}\right)\times\pi\times(17.5)^2\right\}\text{ m}^2$$

$$= \pi\times\left(\frac{35}{2}\right)^2\text{ m}^2 = \frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}\text{ m}^2 = 962.5 \text{ m}^2$$



(ii) Let DL be prependicular drawn from D on AB. Then,

AL = AB – BL = AB – CD = (75 – 50) m = 25 m

In ΔALD, we have

$\tan 60^\circ = \frac{DL}{AL} \Rightarrow \sqrt{3} = \frac{DL}{25} \Rightarrow DL = 25\sqrt{3}$ m

$\therefore$ Area of trapezium ABCD

= 1/2 (AB + CD) × DL

= 1/2 (75 +50) × $25\sqrt{3}$ m^2

= 1562.5 × 1.732 m^2 = 2706.25 m^2

Hence,

Area of the remaining portion

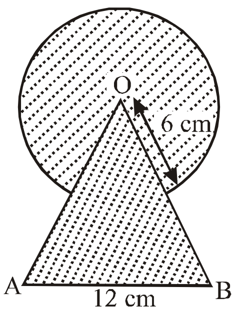
= Area of trapezium ABCD – Area of 4 sectors

= 2706.25 m^2 – 962.5 m^2 = 1743.75 m^2

Ex.45 Find the area of the shaded region in Fig. where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

Sol. We have,

Required Area = Area of ΔOAB + Area of the circle – Area of a sector of a circle of radius 6 cm and of angle 60º



$\Rightarrow$ Required Area

$$= \left\{\frac{\sqrt{3}}{4} \times 12^2 + \pi \times 6^2 \frac{60}{360} \times \pi \times 6^2\right\} \text{cm}^2$$

$$= (36\sqrt{3} + 36\pi - 6\pi)\,\text{cm}^2$$

$$= \left(36\sqrt{3} + 30 \times \frac{22}{7}\right)\text{cm}^2 = \left(\frac{660}{7} + 36\sqrt{3}\right)\text{cm}^2$$

Ex.46 On a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. Find the area of the design (shaded region).

Sol. In ΔOBD, we have

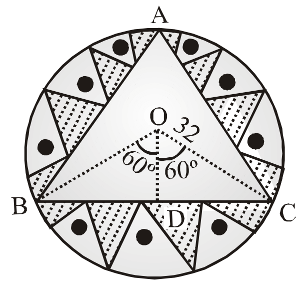
$\cos 60^\circ = \frac{OD}{OB}$ and $\sin 60^\circ = \frac{BD}{OB}$

$\Rightarrow \frac{1}{2} = \frac{OD}{32}$ and $\frac{\sqrt{3}}{2} = \frac{BD}{32}$

$\Rightarrow$ OD = 16 and BD = $16\sqrt{3}$

$\Rightarrow$ BC = 2 BD = $32\sqrt{3}$

Area of the shaded region



= Area of the circle – Area of ΔABC

$$= \left\{\pi \times 32^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2\right\} \text{cm}^2$$

$$= \left\{\frac{22}{7} \times 32 \times 32 - 768\sqrt{3}\right\} \text{cm}^2$$

$$= \left\{\frac{22528}{7} - 768\sqrt{3}\right\} \text{cm}^2$$

Ex.47 In Fig. AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm,find the area of shaded region.

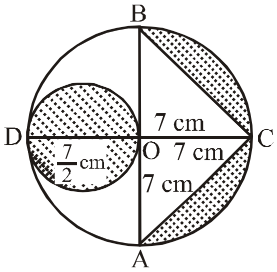
Sol. We have,

Area of the shaded region

= (Area of circle with OD (= 7 cm) as diameter)

+ Area of semi-circle with AB as diameter

– Area of ΔABC



$$= \pi \times \left(\frac{7}{2}\right)^2 + 1/2 \times \pi \times (7)^2 - 1/2 \times AB \times OC$$

$$= \left\{\frac{\pi}{4} \times 49 + \frac{\pi}{2} \times 49 - \frac{1}{2} \times 14 \times 7\right\} \text{cm}^2$$

$$= \left(\frac{3\pi}{4} \times 49 - 49\right) \text{cm}^2 = \left(\frac{3}{4} \times \frac{22}{7} \times 49 - 49\right) \text{cm}^2$$

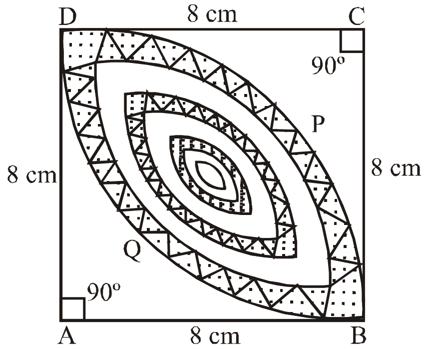
$$= \frac{231 - 98}{2} \ \text{cm}^2 = 66.5 \text{ cm}^2$$

Ex.48 Calculate the area of the designed region in Fig. common between two quadrants of circles of radius 8 cm each.

Sol. We have,

Area of the designed region

= 2(Area of quadrant ABCD – Area of ΔABD)



$$= 2\left\{\frac{\pi}{4} \times (8)^2 - \frac{1}{2} \times 8 \times 8\right\} \text{cm}^2$$

$$= 2\left\{\frac{22}{7} \times \frac{1}{4} \times 64 - 32\right\} \text{cm}^2$$

$$= 2\left\{\frac{22 \times 16}{7} - 32\right\} \text{cm}^2$$

$$= 2\left(\frac{352 - 224}{7}\right) \text{cm}^2 = \frac{256}{7} \text{cm}^2$$

IMPORTANT POINTS TO BE REMEMBERED

- For a circle of a radius r, we have

  (i) Circumference = $2\pi r$

  (ii) Area = πr^2

  (iii) Area of semi-circle = $\frac{\pi r^2}{2}$

  (iv) Area of a quadrant = $\frac{\pi r^2}{4}$

- If R and r are the radii of two concentric circles such that R > r then area enclosed by the two circles

  $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

- If a sector of a circle of radius r contains an angle of θ^o. Then,

  (i) Length of the arc of the sector

  $= \frac{\theta}{360} \times 2\pi r$

  $= \frac{\theta}{360} \times$ (Circumference of the circle)

  (ii) Perimeter of the sector = $2r + \frac{\theta}{360} \times 2\pi r$

  (iii) Area of the sector

  $= \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times$ (Area of the circle)

  (iv) Area of the segement

  = Area of the corresponding sector

  – Area of the corresponding triangle

  $= \frac{\theta}{360} \times \pi r^2 - r^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$

  $= \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r$