

MECHANICAL PROPERTIES OF SOLIDS

HOOKE'S LAW

Hooke's law also referred to as the law of elasticity was discovered by an English scientist named Robert Hooke in the year 1660. Hooke's law basically states that "when an object has a relatively small deformation the size of the deformation is directly proportional to the deforming load or force."

Besides, Hooke's law is a great example to describe the property of elasticity – which is a tendency of an object or material to be restored to its original shape after some form of distortion. The ability to return to a normal or original shape can be described as a "restoring force". Explained more clearly by Hooke's Law, this restoring force is said to be proportional to the "stretch" that is experienced.

Hooke's law can be usually taken as a first-order linear approximation only to the response that springs and other elastic bodies offer when force is applied. The law will eventually fail after certain conditions. It fails usually when the forces exceed some limit, the material reaches its minimum compressibility size or its maximum stretching size. Alternatively, there will also be some permanent deformation or change of state once the thresholds are crossed. In fact, some of the materials will mostly start deviating from Hooke's law much before those limits are met.

Meanwhile, we can also say that Hooke's law is an accurate approximation for most solid bodies. However, this will mainly depend on the forces and deformations wherein they should be very small. As a result, Hooke's law is used in many different branches of science and engineering. It is also the foundation of many disciplines such as molecular mechanics, seismology and acoustics. This law is further the most fundamental principle behind the galvanometer, spring scale, manometer and balance wheel of a mechanical clock.

Hooke's Law Statement

When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called linearly elastic material. In this case, stress is directly proportional to strain.

OR

You can say that “for small deformation, stress is directly proportional to strain”

Therefore, in simple terms, Hooke's law states that the strain in a solid is proportional to the applied stress within the elastic limit of that solid.

Hooke's Law Equations

Hooke's equation holds in many situations where an elastic body is deformed. The instances may include the wind blowing on a tall building or a musician playing the guitar. The equation is said to be linear-elastic or Hookean usually for an elastic body or material.

Hooke's Law Equation in Terms of Stress and Strain

According to this law, within the elastic limit, stress is proportional to the strain. Thus, Hooke's Law equation can be expressed in terms of stress and strain;

Stress \propto Strain or stress / strain = constant = E

Stress = Young's modulus of elasticity \times Strain

$$\sigma = E \varepsilon$$

Where,

- σ is the stress,
- E is the modulus of elasticity also known as Young's modulus of elasticity,
- ε is the strain.

In SI units, the spring constant k , and each element of the tensor κ , is measured using units such as newtons per meter (N/m), or kilograms per second squared (kg/s^2).

For continuous media, each element is therefore measured in units of pressure, namely pascals (Pa, or N/m^2 , or $\text{kg}/(\text{m}\cdot\text{s}^2)$). The elements of the strain tensor ϵ are also expressed in units of pressure.

For Linear Springs

Mathematically, Hooke's law states that the applied force F equals a constant k , into the displacement or change in length x , or $F = kx$. The value of k depends not only on the kind of elastic material under consideration but also on its dimensions and shape.

All materials exhibit some degree of elasticity. Due to this elastic property, an excellent concept of restoring force comes into the picture. We can measure this elasticity property in the form of a restoring force. This restoring force opposes the deformation force and tries to maintain the original dimensions of the material.

Now, consider the elasticity in only one dimension. To verify Hook's Law on this spring-mass system we need to find the relation between the restoring force and stretch (elongation) for a spring. As we know, the restoring force is proportional to the magnitude of the deformation. This restoring force can be written mathematically as;

$$F = - kx.$$

This expression for Hooke's Law is also known as the spring constant formula.

Here,

- F is restoring force.
- x represents the magnitude of the distortion or displacement from equilibrium as exhibited in the stretching of a spring or rubber band.
- k is the proportionality constant, also known as the spring constant.

Note

Here the direction of the force is in the direction opposite that of the displacement, so consider minus sign.

General Scalar Springs

Hooke's spring law is also applicable to different types of elastic material or object with random complexity. However, this will depend on whether the stress and deformation can be represented by a single number. This number can be either positive and negative.

For instance, if we take a rubber block and attach it to two plates (kept in parallel) is distorted by shearing. So, the shearing force F_s and the sideways displacement of the plates x follow or adhere to Hooke's law (small deformations).

Vector Formulation

If we stretch a helical spring or compress it along its axis then the restoring force, as well as the resulting elongation or compression, have the same direction. As such, when F_s and x are defined as vectors then Hooke's equation will still remain valid and it will state that the force vector is the elongation vector multiplied by a fixed scalar.

Applications of Hooke's Law

- The applications of Hooke's Law is as given below:
- Most commonly, in everyday life, Hooke's Law is applied in springs because of their elasticity.
- They are used not only in the engineering field but also used in the field of medical science.
- It is used in breathing (lungs), skin, spring beds, diving boards and cars suspension systems.
- It is used as a fundamental principle behind the manometer, spring scale and balance wheel of the clock.
- It is also used as the foundation for seismology, acoustics and molecular mechanics.

Disadvantages of Applying Hooke's Law

The disadvantages of Hooke's Law is as follows:

- Hooke's Law is applied only in the elastic region.
- Hooke's Law gives accurate results only for solid bodies if the forces and deformations are small.

Hooke's Law is not a universal law.

While we have learnt so much about Hooke's Law, let us also understand the subsequent terms which can be used inside the mechanical properties of solids. Let us quickly go through them.