MECHANICAL PROPERTIES OF SOLIDS

ELASTIC MODULI

Elastic Constants

- Elastic constants are those constants which determine the deformation produced by a given stress system acting on the material.
- Elastic constants are used to determine engineering strain theoretically.
- They are used to obtain a relationship between engineering stress and engineering strain.
- For a homogeneous and isotropic material, the number of elastic constants are 4.

Types of Elastic Constants

- Young's modulus or modulus of Elasticity (E)
- Shear modulus or modulus of rigidity (G)
- Bulk modulus (K)
- Poisson's Ratio (μ)

In this article, we discuss only on the first type that is Young's modulus or modulus of Elasticity (E)

Relationship Between Elastic Constants

- $E = 2G\,(1+\mu)$
- $E = 3\,K\,(1-2\mu)$
- $E = 9KG\,/\,G+3K$

Hope you understood the relation between Young's modulus and bulk modulus k and modulus of rigid

Value of any constant is always greater than or equal to 0. Negative sign only shows the direction.

Definition of Modulus of Elasticity

As per Hooke's law, up to the proportional limit, "for small deformation, stress is directly proportional to strain."

Mathematically, Hooke's Law expressed as:

Stress α Strain

$$\sigma = E\,\varepsilon$$

In the formula as mentioned above, "E" is termed as Modulus of Elasticity.

σ is the Stress, and ε denotes strain.

We can write the expression for Modulus of Elasticity using the above equation as,

$$E = (F*L) / (A * \delta L)$$

So we can define modulus of Elasticity as the ratio of normal stress to longitudinal strain.

Unit of Modulus of Elasticity

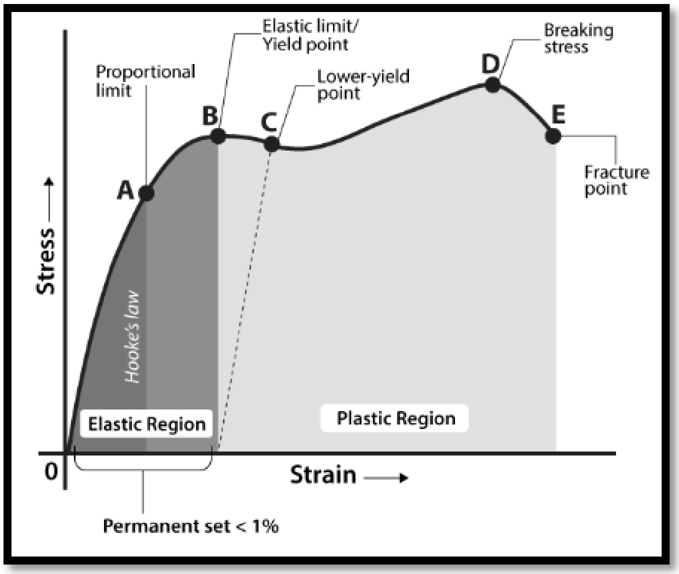
The unit of normal Stress is Pascal, and longitudinal strain has no unit. Because longitudinal strain is the ratio of change in length to the original length. So the unit of Modulus of Elasticity is same as of Stress, and it is Pascal (Pa). We use most commonly Megapascals (MPa) and Gigapascals (GPa) to measure the modulus of Elasticity.

1 MPa = Pa

1 GPa = Pa

Measure Young's Modulus or Modulus of Elasticity

Let us take a rod of a ductile material that is mild steel. Now do a tension test on Universal testing machine. After the tension test when we plot Stress-strain diagram, then we get the curve like below.



From the curve, we see that from point O to B, the region is an elastic region. After that, the plastic deformation starts. The point A in the curve shows the limit of proportionality. For this curve, we can write the value of Modulus of Elasticity (E) is equal to the slope of Stress-strain curve up to A.

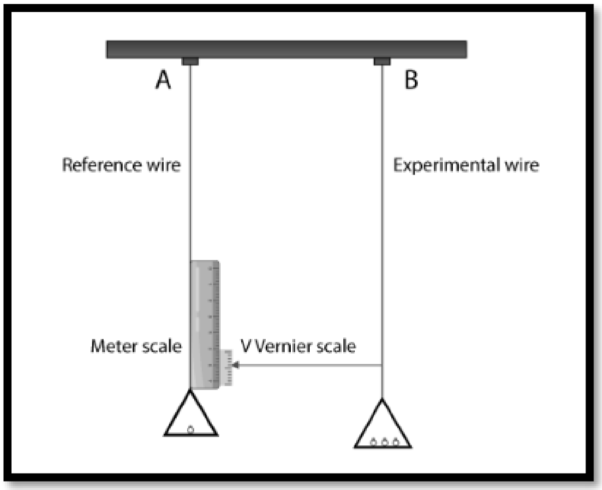
If the value of E increases, then longitudinal strain decreases, that means a change in length decreases.

Here are some values of E for most commonly used materials.

- Mild steel- E = 200 GPa
- Cast iron E = 100 GPa
- Aluminum E = 200/3 GPa

Determination of Young's Modulus of the Material of a Wire

Make an experimental arrangement as shown in the figure to determine the value of Young's modulus of a material of wire under tension.



Take two identical straight wires (same length and equal radius) A and B. Now fix its end from a fixed, rigid support. The wire A is the reference wire, and it carries a millimetre main scale M and a pan to place weight. The wire B is the experimental wire. It also carries a pan in which known weights are placed. At the bottom of the wire, B attaches a vernier scale V.

Now, after putting the weight in the pan connected to B, it exerts a downward force. In the influence of this downward force (tensile Stress), wire B get stretched. This elongation (increase in length) of the wire B is measured by the vernier scale. The reference wire A is used to compensate for any change in length that may occur due to change in room temperature. Initially, give a small load to both the wires A and B so that both be straight and take the and Vernier reading. Now increase the load gradually in wire B and note the vernier reading. The difference between these two vernier readings gives the change in length produced in the wire.

Let initial radius and length of the wire B is r and L respectively,

Then the cross-sectional area of the wire would be pr^2

Let M be the mass that is responsible for an elongation DL in the wire B.

Then the applied force is equal to Mg, where g is the acceleration due to gravity.

From the equation,

$E= (F/A) / (DL/L)$

$= (F \times L) / (A \times DL)$

The Young's modulus of the material of the experimental wire B is given by;

$Y = \sigma/\varepsilon$

$Y = (F/A)/(\Delta L/L)$

$Y= (F \times L) /(A \times \Delta L)$

$Y= (Mg \times L) /(A \times \Delta L)$

Interesting facts about Modulus of Elasticity

- Modulus of Elasticity and Young's Modulus both are the same. The modulus of elasticity is constant.
- Robert Hooke introduces it. Robert Hooke (1635 – 1703) is the Early Scientist Worked on Applied Mechanics.
- Whereas Young's modulus is denoted as "E" in 1807 by Thomas Young. It is explained in "Course of Lectures on Natural Philosophy and the Mechanical Arts" which is written by Thomas Young. He did detailed research in Elasticity Characterization.
- According to the Robert Hook value of E depends on both the geometry and material under consideration. For find out the value of E, it is required physical testing for any new component.
- Thomas Young said that the value of E depends only on the material, not its geometry". Thus he made a revolution in engineering strategies.
- A small piece of rubber and a large piece of rubber has the same elastic modulus.
- Modulus of Elasticity is also known as the tensile modulus or Elastic modulus.
- It is a fundamental property of every material that cannot be changed. It dependents upon temperature and pressure, however.

- The Elastic Modulus is the measure of the stiffness of a material. In other words, it is a measure of how easily any material can be bend or stretch.
- It is the slope of stress and strain diagram up to the limit of proportionality.

What are its applications

- It is used in engineering as well as medical science.
- You can use the elastic modulus to calculate how much a material will stretch and also how much potential energy will be stored.
- The elastic modulus allows you to determine how a given material will respond to Stress.
- Elastic modulus is used to characterize biological materials like cartilage and bone as well.