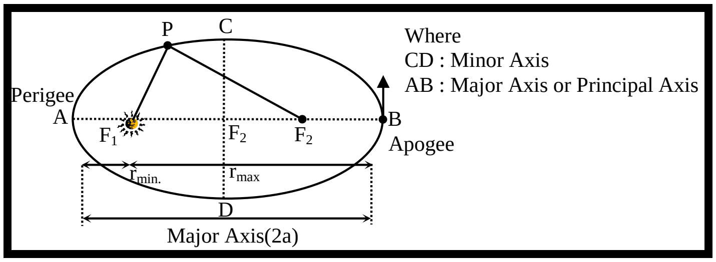
GRAVITATION

KEPLER'S LAWS

Kepler's laws of planetary motion:

First law

Every planet revolves around the sun in an elliptical orbits and sun is at one focus.



Ellipse

When a particle moves with respect to two fixed points in such way that the sum of the distances from these two points is always constant then the path of the particle is ellipse. And the two fixed points are called **focus** points.

According to Figure: -

$$PF_1 + PF_2 = AF_1 + AF_2 = BF_1 + BF_2 = \text{constant}$$

But in ellipse $AF_1 = BF_2$ (minimum distance from both focus is same)

$$PF_1 + PF_2 = BF_2 + AF_2 = BF_1 + AF_1 = 2a$$

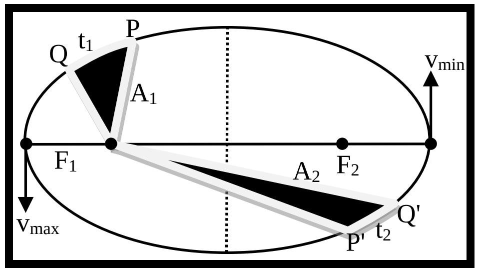
$$r_1 + r_2 = r_{min} + r_{max} = 2a$$

$$\boxed{a = \frac{r_1 + r_2}{2} = \frac{r_{min} + r_{max}}{2}} \text{ (Mean distance)}$$

Second Law

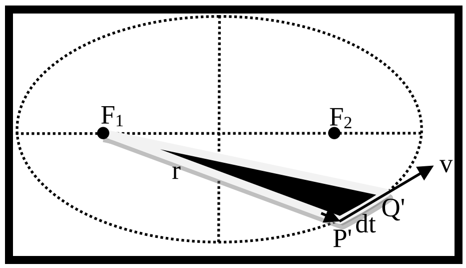
According to this law the radius vector joining sun and planet sweeps out equal area in equal interval of time.

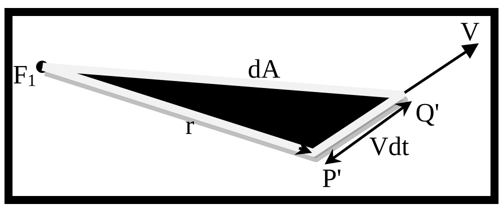
If $t_1 = t_2$ then $A_1 = A_2$



Note

When a planet reaches nearest to sun then speed of planet is maximum and when it is farthest from sun then speed is minimum.





Area covered by planet in short interval of time is

$$dA = \frac{1}{2} \times \text{base} \times \text{height} \Rightarrow dA = \frac{1}{2} \times r \times Vdt$$

$\boxed{\frac{dA}{dt} = \frac{1}{2} rv}$ or Areal speed of planet is always constant $\boxed{\frac{dA}{dt} = \frac{1}{2} rv = \text{constant}}$

Important Points

(1) We know that in planetary motion

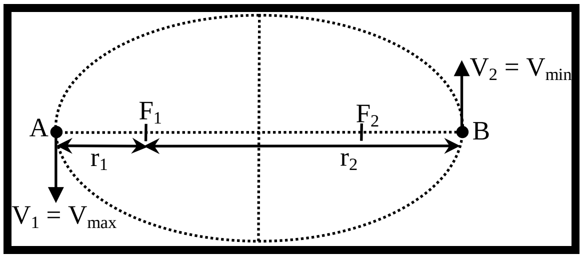
$$\frac{1}{2} rV = \text{constant} \Rightarrow \frac{1}{2} rV \times \frac{m}{m} = \text{constant}$$

$$\frac{mVr}{2m} = \text{constant} \Rightarrow mvr = \text{constant}$$

$$\boxed{L = \text{constant}}$$

So in planetary motion, angular momentum of the planet is always conserved **(COAM)**

(2) Apply COAM between points (A) & (B)



$L_A = L_B$

$mV_{max.}\, r_{min.} = m.\, V_{min.}\, r_{max.}$

$\boxed{V_{max.} r_{min.} = V_{min.} r_{max.}}$

Third law:- time period law

Square of the time period is directly proportional to the cube of the mean distance (semi major axis)

$T^2 \propto r^3$

$T^2 = K_s r^3$ ... (1)   Where K_s : Kepler's constant

$K_s = \dfrac{4\pi^2}{GM_s} = 2.97 \times 10^{-17}\ sec^2/m^3$ (M_s : Mass of Sun)

Substituting the value of K_s in above eq. (1)

$$\boxed{T^2 = \frac{4\pi^2}{GM_s}.r^3}$$

Standard M.T.R. for Questions: -

(1) Time period of earth on its own axis is $T_e = 24$ hrs.

(2) Time period of earth about sun is $T = 365$ days

(3) The period of moon around earth is $T_m = 27.3$ days which is also called **"Lunar month"**

(4) Time period of Geo stationary satellite is 24 hrs.

Q. The mean radius of the earth's orbit around the sun is 1.5×10^{11} meter. The mean radius of the orbit of mercury around the sun is 6×10^{10} metre. The mercury will rotate around the sun is:-

Ans. $\frac{1}{4}$ years.

Q. A satellite moves in a circular orbit around the earth. The radius of this orbit is one half that of moon's orbit. The satellite completes one revolution in

Ans. $2^{-3/2}$ Lunar month

Q. If earth describes an orbit round the sun of double its present radius, the year on earth will be of-

Ans. $365 \times 2\sqrt{2}$ days