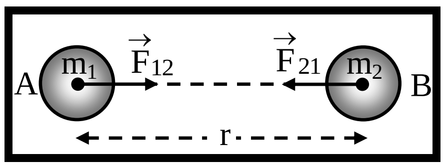
GRAVITATION

INTRODUCTION OF GRAVITATION

NEWTON'S LAW OF GRAVITATION

It states that everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them



According to Newton's law

$F \propto m_1 m_2 \ldots\ldots (1)$

$F \propto \frac{1}{r^2} \ldots\ldots\ldots (2)$

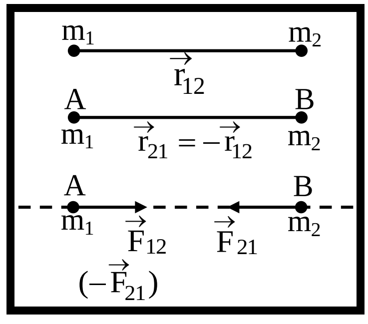
From (1) & (2) we get

$F \propto \frac{m_1 m_2}{r^2}$

$$F = -\frac{G m_1 m_2}{r^2}$$

where G = Universal gravitational constant

Vector form of Newton's law of Gravitation



Let

- $\vec{r}_{12}$ = displacement vector from A to B
- $\vec{r}_{21}$ = displacement vector from B to A
- $\vec{F}_{21}$ = Gravitational force exerted on B by A
- $\vec{F}_{12}$ = Gravitational force exerted on A by B

Gravitational Force

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21} = -\frac{Gm_1m_2}{r_{21}^3}\vec{r}_{21} = +\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} = +\frac{Gm_1m_2}{r_{12}^3}\vec{r}_{12}$$

Negative sign shows that

(i) the direction of $\vec{F}_{12}$ is opposite to that $\hat{r}_{21}$

(ii) the gravitational force is attractive in nature Similarly

$$\vec{F}_{21} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$

Conclusion:

$$\vec{F}_{12} = -\vec{F}_{21}$$

The gravitational force between two bodies are equal in magnitude and opposite in direction.

Examples :

Ex. Two spherical balls of mass 10 kg each are placed 100 m apart. Find the gravitational force of attraction between them.

Sol. $F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times10\times10}{(100)^2}$

$= 6.67 \times 10^{-13}$ N

Ex. Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of heavier particle.

Sol. The force of gravitation exerted by one particle on another is

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{(0.5)^2} = 5.3\times10^{-10}\text{N}$$

Acceleration of heavier particle

$$= \frac{F}{m_2} = \frac{5.3\times10^{-10}}{2} = 2.65\times10^{-10}\text{ms}^{-2}$$

Ex. The value of universal gravitational constant G depends upon.

(1) nature of material of two bodies

(2) medium between the two bodies

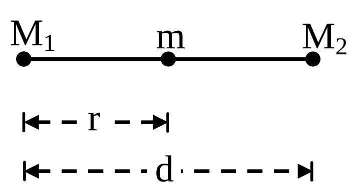
(3) acceleration of two bodies

(4) none of the above

Sol. Because the Gravitational constant G is a universal constant. So answer is (4)

Ex. Two stationary particles of masses M_1 and M_2 are at a distance 'd' apart. A third particle lying on the line joining the particles, experiences no resultant gravitational forces. What is the distance of this particle from M_1.

Sol. The force on m towards M_1 is



$$F_1 = \frac{GM_1m}{r^2}$$

The force on m towards M_2 is

$$F_2 = \frac{GM_2m}{(d-r)^2}$$

According to question net force on m is zero

$F_1 = F_2$

$$\frac{GM_1m}{r^2} = \frac{GM_2m}{(d-r)^2} \Rightarrow \left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1}$$

$$\frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}} \Rightarrow r = d\left(\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right)$$