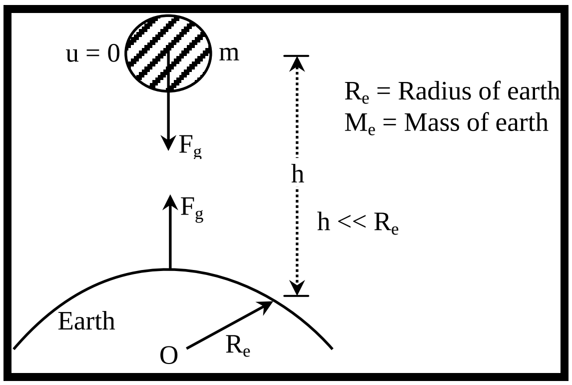
GRAVITATION

ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH

ACCELERATION DUE TO GRAVITY



$$F_g = ma \Rightarrow \frac{GM_e m}{R_e^2} = m.a_g$$

$$\boxed{a_g = g = \frac{GM_e}{R_e^2}} \quad \boxed{GM_e = gR_e^2}$$

Important Points:

(1) In form of density $g = \frac{GM_e}{R_e^2} = \frac{G}{R_e^2} \times \frac{4}{3}\pi R_e^3 \times \rho$

$$\boxed{g = \frac{4}{3}\pi G.R_e.\rho}$$

If ρ is constant then $\boxed{g \propto R_e}$

(2) If M is constant $g \propto \frac{1}{R^2}$

% form of variation in 'g' (upto 5%)

$$\boxed{\frac{\Delta g}{g} = -2.\frac{\Delta R_e}{R_e}}$$

(3) If mass (M) and radius (R) of a planet, if small change is occurs in (M) and (R) then g $= \frac{GM}{R^2}$

$$\boxed{\frac{\Delta g}{g} = \frac{\Delta M}{M} - 2.\frac{\Delta R_e}{R_e}}$$

If R is constant $\frac{\Delta g}{g} = \frac{\Delta M}{M}$

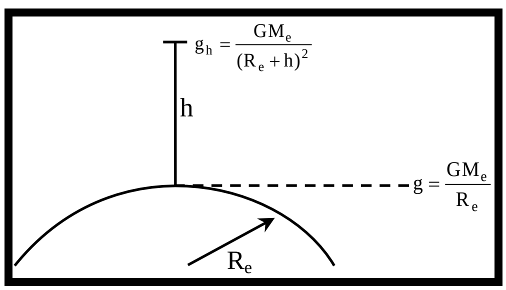
If M is constant $\boxed{\frac{\Delta g}{g} = -2.\frac{\Delta R_e}{R_e}}$

Q. If radius of earth is 1% shrink then what will be change in acceleration due to gravity? (Mass of the earth constant.)

Ans. + 2% (increased)

VARIATION IN 'g' :

(1) Due to Altitude (height):-



$$\frac{g_h}{g} = \frac{R_e^2}{(R_e+h)^2} = \frac{R_e^2}{R_e^2[1+h/R_e]^2}$$

$$g_h = g\left[1+\frac{h}{R_e}\right]^{-2}$$

Expand by binomial theorem

$(1 + x)^n = 1 + n. x + \frac{n.(n-1)}{2!} . x^2 + ...$

if $x << 1$ higher power terms are negligible and $(1 + x)^n = 1 + nx$

$h << R_e$ higher power terms are negligible

$$\left(1+\frac{h}{R_e}\right)^{-2}=\left(1-\frac{2h}{R_e}\right)$$

$$\boxed{g_h = g\left[1-\frac{2h}{R_e}\right]}$$

Note [This formula is valid if h is upto 5% of earth radius. (320 km. from earth surface)]

If h is greater than 5% of earth radius we use

$$\boxed{g_h = \frac{GM}{(R_e+h)^2}}$$

QUESTIONS:

Q. At which height from earth surface, acceleration due to gravity is decreased by 1%

Sol. $\Delta g = g - g_h = g - g\left[1-\frac{2h}{R_e}\right]$

$\Delta g = \frac{2gh}{R}$ $\boxed{\therefore \frac{\Delta g}{g}=\frac{2h}{R_e}}$ valid upto 5% of earth radius

$\frac{1}{100}=\frac{2h}{R_e} \Rightarrow \frac{1}{100}=\frac{2h}{6400}$

h = 32 km. from earth surface.

Q. Find the percentage decrease in the weight of the body when taken to a height of 16 km. above the surface of earth (radius of earth is 6400 km.)

Ans. 0.5%

Q. What is the value of acceleration due to gravity at height equal to half the radius of earth from surface of earth. [take g = 10 m/s^2 at earth surface]

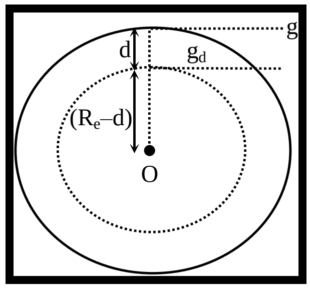
Ans. $\frac{40}{9}\,\text{m/s}^2$

Q. AT which height from the earth surface acceleration due to gravity will be 1/4 of its value at earth's surface.

Ans. $h = R_e$

(2) Due to depth:-

Density of earth remains same throughout



At earth surface

$g = \frac{4}{3}\pi G.\, R_e.\, \rho$..........(1)

At depth d inside the earth

$g_d = \frac{4}{3}\pi G(R_e - d)\rho$..........(2)

from (1) & (2) we get

$$\frac{g_d}{g} = \frac{R_e - d}{R_e} = \left[1 - \frac{d}{R_e}\right]$$

$\boxed{g_d = g\left[1 - \frac{d}{R_e}\right]}$ valid for 100% depth

Special Case:-

$\Delta g_d = g - g_d =$ decrement in depth with depth $= g - g\left[1 - \frac{d}{R_e}\right]$

$\Delta g_d = \frac{gd}{R_e}$ $\boxed{\frac{\Delta g_d}{g} = \frac{gd}{R_e}}$

QUESTIONS:

Q. At which depth from earth surface, acc^n due to gravity is decreased by 1%

Sol. $\frac{\Delta g_d}{g} = \frac{d}{R_e}$

$\frac{1}{100} = \frac{d}{6400}$ $d = 64$ km.

Q. At which height above earth's surface value of 'g' is same as in a mine 100 km deep.

Ans. 50 km.

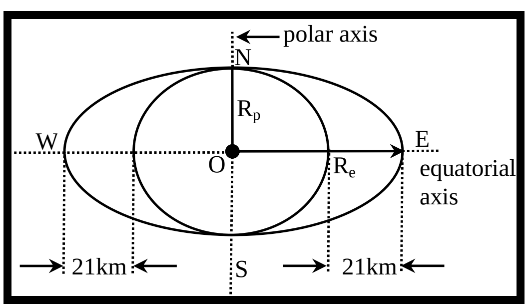
Q. How much below the surface does the acceleration due to gravity becomes 70% of its value on the surface of earth.

Ans. $d = \frac{3}{10} R_e$

Q. At which depth from earth surface does the acceleration due to gravity becomes 1/4 times of g.

Ans. $d = \frac{3}{4} R_e$

(3) Due to shape of the Earth:-



$R_p < R_e$

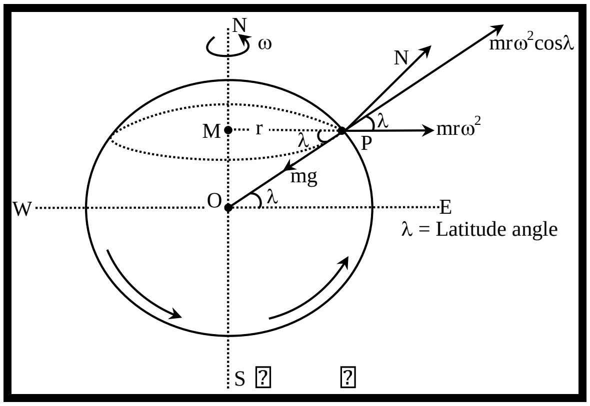
$R_e = R_p + 21$ km.

$g_p = \frac{GM_e}{R_p^2}$ and $g_e = \frac{GM_e}{(R_p + 21)^2}$

$g_e < g_p$

$\boxed{g_p - g_e = 0.02 m/s^2}$

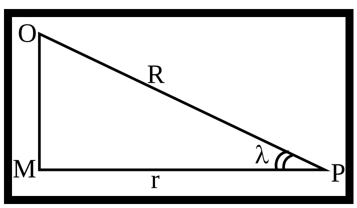
4) Due to Rotation of the earth:-



Net force on particle mg' = mg – mr ² cos

or g' = g – rω² cos λ ....(1)

From fig. In OMP



$\boxed{r = R_e \cos\lambda}$

Substituting value of r in eq. (1) we get

$\boxed{g' = g - R_e\omega^2 \cos^2\lambda}$

Important Points:-

(1) If latitude angle λ = 0. It means at equator.

$\boxed{g'_{min.} = g_e = g - R_e\omega^2}$ ... (1)

(2) If latitude angle λ = 90º, it means at poles.

$g'_{max.} = g_p = g$ ... (2)

from (1) and (2) equation.

$g_p > g_e$

then $\Delta g_{rot.} = g_p - g_e = 0.03\ m/s^2$ only due to rotation

(3) $\Delta g_{total} = g_p - g_e = (0.05\ m/s^2)$

$\to 0.02\ m/s^2$ (Due to shape 40%)

$\to 0.03\ m/s^2$ (Due to rotation 60%)

(4) If rotation of earth suddenly stops then acc^n due to gravity is increases at all places on earth except the poles.

(5) **Condition of weightlessness:-**

If apparent weight of body is zero then angular speed of earth can be calculated as

$mg' = mg - mR_e\omega^2\cos^2\lambda$

$0 = mg - mR_e\omega^2\cos^2\lambda$ or $\omega = \frac{1}{\cos\lambda}\sqrt{\frac{g}{R_e}}$

But at equator $\lambda = 0^\circ$

$\omega = \sqrt{\frac{g}{R_e}} = \frac{1}{800}$ rad/sec. = 0.00125 rad/sec.

$= 1.25 \times 10^{-3}$ rad/sec. [17 times of present angular speed]

Time period of earth's rotation $T = \frac{2\pi}{\omega} = 1.4$ hr.