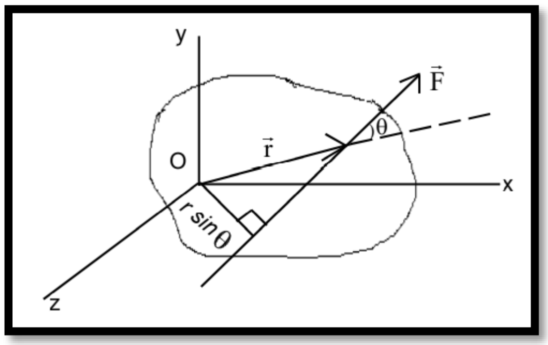
SYSTEM OF PARTICLES AND ROTATIONAL MOTION

TORQUE AND ANGULAR MOMENTUM

TORQUE



(a) The torque of force F about the point O is equal to the product of force and perpendicular distance of line of action of force from point.

$\tau = 1/4$ Force $^{1/2} \times 1/4$ Perpendicular distance of line of action of force from point O)

$= \mathrm{Frsin}\,\theta = (\mathrm{Fsin}\,\theta)\mathrm{r}$

$= 1/4$The component of force perpendicular

to position vector) $\times$ (Position vector)

$\tau = \mathrm{Frsin}\ \theta, \mathrm{rsin}\ \theta$ is known as lever arm

(b) **Unit:**

In M.K.S = N-m

In C.G.S = dyne-cm

(c) **Dimension:**

ML^2T^{-2}

(d) In vector form $\vec{\tau} = \vec{r} \times \vec{F}$

$= rF\sin\ \theta\hat{n}$, where θ is angle between $\vec{r}$ and $\vec{F}$.

and $\hat{n}$ is unit vector perpendicular to the plane

of $\vec{r}$ and $\vec{F}$

(e) Torque is a vector quantity, whose direction is perpendicular to the plane of force and position vector and its direction is given by right hand screw rule

(f) If the torque rotates the body in antilock wise direction, the torque is positive and if the torque rotates the body in clock-wire direction, the torque will be negative

(g) If a body is acted upon by more than one force, the total torque is the vector sum of each torque

$\vec{\tau} = \vec{\tau}_{1++} + \vec{\tau}_2 + \vec{\tau}_2 + \cdots \ldots \ldots + \vec{\tau}_n$

(h) $\tau = I\alpha$

I - Moment of inertia with respect to axis of rotation.

α - Angular acceleration with respect to axis of rotation

τ - Torque of force which is causing the rotational motion

(i) $\vec{\tau} = \frac{d\vec{J}}{dt}$, where $\vec{J}$ is angular momentum

(j) The more is the value of r, the more will be torque and easier to rotate the body.

i) The handle of screw driver is taken thick.

ii) In villages the handle of flour-mill is placed near the circumference.

iii) The handle of handpump is kept-long.

iv) The ranch used for opening the tap, is kept-long.

(k) Work done by torque $= \int_{\theta_1}^{\theta_2} \tau d\theta$

$= \text{Torque} \times \text{angular displacement}$

Ex. Given that, $\vec{r} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} + 6\hat{k}$. The magnitude of torque will be-

(1) $\sqrt{405}$ N.m (2) $\sqrt{410}$ N.m

(3) $\sqrt{504}$ N.m (4) $\sqrt{510}$ N.m

Sol. (3) We know that, $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = (2\hat{i} + 3\hat{j}) \times (2\hat{i} + 6\hat{k})$

$12(-\hat{j}) + 6(-\hat{k}) + 18\hat{i}$

$-12\hat{j} - 6\hat{k} + 18\hat{i}$

[Note : $\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = k, \hat{j} \times \hat{i} = -k$ etc]

Now, $|\vec{\tau}| = \sqrt{(-12)^2 + (-6)^2 + (18)^2}$

$\sqrt{144 + 36 + 324} = \sqrt{504}$

Ex. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 seconds. The magnitude of this toque is-

(1) $3A_0/4$ (2) A_0

(3) $4A_0$ (4) $12A_0$

Sol. (1) $\Delta J = 4A_0 - A_0 = 3A_0$ and $\Delta T = 4$

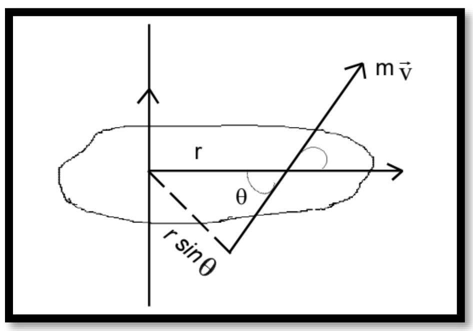
$$\tau = \frac{\Delta J}{\Delta T} = \frac{3A_0}{4}$$

ANGULAR MOMENTUM

(a) The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum.

(b) It is a vector quantity, which is often represented by $\vec{L}$ or $\vec{J}$

(c) Angular momentum $\vec{J} = \vec{r} \times \vec{P}$



$= \vec{r} \times (m\vec{v}) = m\left(\vec{r} \times \vec{v}\right)$

Or $\vec{J} = rp\sin\theta\hat{n} = mvr\sin\theta\hat{n}$

θ is angle between $\vec{r}$ and $\vec{v}$, $\hat{n}$ is unit vector perpendicular to the plane of $\vec{r}$ and $\vec{v}$

(d) The direction of angular momentum is perpendicular to the plane of $\vec{r}$ and $\vec{v}$ and it is given by right hand screw rule.

(e) $J = mvr\ \sin\ \theta$

Cases:

a. If $\theta = 0$, $J=0^{1/4\text{Minimum}^{1/2}}$

b. If $\theta = 90°$, $J = mvr^{1}$/ Maximum 1/2

$= (mr^2)\omega \because v = r\omega$

(f) Unit: J. second, kg m^2/s, kg m^2 rad/sec

(g) Dimension: $[M^1L^2T^{-1}]$.

(h) If direction of rotation is anticlockwise, angular momentum is taken positive and if direction of rotation is clockwise, angular momentum is taken negative.

(i) The angular momentum of a system of particle's is equal to the vector sum of angular momentum of each particle.

$\vec{j} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \cdots.$

(j) Relation between angular momentum and angular velocity $J = I\omega$

I - Moment of inertia with respect to axis of rotation

ω- Angular velocity due to angular momentum

J - The moment of momentum which is causing rotational motion.

(k) The rate of change of angular momentum is equal to the torque applied on the body.

$$\vec{\tau} = \frac{d\vec{J}}{dt}$$

(l) In rotational motion angular momentum has equal importance as linear momentum in linear motion

(M) If torque acting of a particle is zero then

$\vec{\tau} = 0 \Rightarrow \frac{d\vec{J}}{dt} = 0$

Which implies that the angular momentum remains conserved when no external force acts on the body

Law of Conservation of Angular Momentum

1. If no external torque is acting upon a body rotating about an axis, then the angular momentum of the body remains constant that is,

   $J = I\omega = \text{constant}$

2. Proof ÷ & We have read above that when a body rotates about an axis under the action of an external torque τ , the rate of change of angular momentum of the body is equal to the torque; that is,

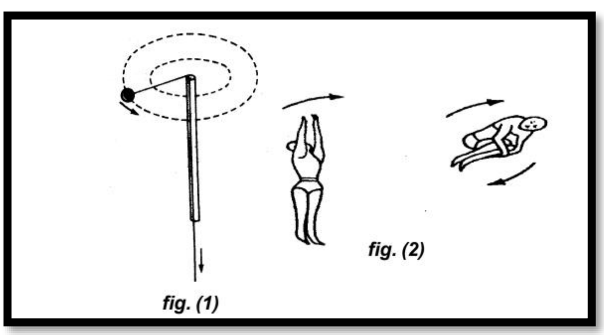
   $\frac{dJ}{dt} = \tau$

   If the external torque is zero $(\tau = 0), \frac{dJ}{dt} = 0$

   $\Rightarrow J = \text{constant}$

   This is the law of conservation of angular momentum.

Examples.



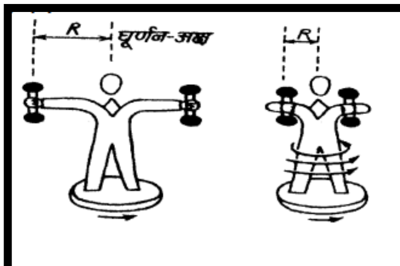


fig. (3)

i) Suppose a ball is tied at one end of a cord whose other end passes through a vertical hollow tube. The tube is held in one hand and the cord in the other. The ball is set into rotation in a horizontal circle. If the cord is pulled down, shortening the radius of the circular path of the ball, the ball rotates faster than before. The reason is that by shortening the radius of the circle, the moment of inertia of the ball about the axis of rotation decreases. Hence, by the law of conservation of angular momentum, the angular velocity of the ball about the axis of rotation increases. [fig. (1)]

ii) When a diver jumps into water from a height, he does not keep his body straight but pulls in his arms and legs towards the centre of his body. On doing so, the moment of inertia I of his body deceases. But since the angular momentum I ω remains constant, his angular velocity ω correspondingly increases. Hence during jumping he can rotate his body in the air. [fig. (2)

iii) In a man with his arms outstretched and holding heavy dumb bells in each hand, is standing at the center of a rotating table. When the man pulls in his arms, the speed of rotation of the table increases. The reason is that on pulling in the arms, the distance R of the dumbbells from the axis of rotation decreases and so the moment of inertia o the man decreases. Therefore, by conservation of angular momentum, the angular velocity increases. [fig. (3)] In the same way, the ice skater and the ballet dancer increase or decrease the angular velocity of spin about a vertical axis by pulling or extending out their limbs.

Ex. A thin circular ring of mass M and radius r is rotating about an axis passing through its center and perpendicular to its plane with a constant angular velocity. Two objects, each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity –

(1) $\frac{\omega(M-2m)}{M+2m}$ (2) $\omega M(M-m)$

(3) $\frac{\omega(M+2m)}{M}$ (4) $\frac{\omega M}{M+2m}$

Sol. (4) As there is no external force acting an the system, so angular momentum will remain conserved. Now

$J = I_1\omega_1 = I_2\omega_2$

$MR^2\ \omega = (MR^2 + mR^2 + mR^2)\omega_2 \Rightarrow$

$$\omega_2 = \frac{M\omega}{M+2m}$$

Ex. In a playground there is a merry go round of mass 120 kg and radius 4 m. The radius of gyration is 3 m. A child of mass 30 kg runs at a speed of 5 m/sec tangent to the rim of the merry go round when it is at rest and then jumps on it. It we neglect the friction; the angular velocity of the merry-go-ground and child will be-

(1) 0.1rad/sec (2) 0.2rad/sec

(3) 0.4rad/sec (4) 0.8rad/sec

Sol. (3) $m_c vr = I\omega = [m_c r^2 + mk^2]\omega$

Given that, r = 4 m, m = 120 kg

$30 \times 5 \times 4 = (120 \times 3^2 + 30 \times 4^2)\ \omega$ and

$m_c = 30$ kg

$$\omega = \frac{600}{1080+480} = 0.4\text{rad/sec}$$

Ex. A body of mass 1.0 kg is rotating on a circular path of diameter 2.0 m at the rate of 10 rotations in 31.4 sec. The angular momentum of the body is- (in kg. m^2/s)

(1) 3 (2) 4

(3) 2 (4) 1

Sol. (3) Mass of the body,m = 1.0 kg

The distance of the body from the axis of rotation,

$r = \frac{2.0}{2} = 1.0$ m

Moment of inertia of the body about the axis of rotation is,

$I = mr^2 = (1).(1)2 = 1$ kg-m^2

Angular velocity of the body, $\omega = 2\pi n$, where n is the number of rotations per/sec

Here, $n = \frac{10}{31.4}$

$\omega = 2 \times 3.14 \times \frac{10}{31.4} = 2\text{rad/s}$

Angular momentum, $J = I\omega = 1 \times 2 = 2$ kg-m^2/s