SYSTEM OF PARTICLES AND ROTATIONAL MOTION

MOTION OF CENTRE OF MASS

Motion of the Centre of Mass

A system consists of n number of particles having masses

$$m_1, m_2, m_3, \ldots m_n$$

and total mass of the system is M. From the definition of centre of mass,

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n \ldots\ldots \ (1)$$

If the mass of each particle of the system remains constant with time, for this system of particles with fixed mass, differentiating the above eq with respect to time we get

$$M\frac{d\vec{r}_m}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt} \ldots\ldots(2)$$

$$M\vec{V}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n \ldots\ldots (3)$$

Here,

$\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_n$ are position vectors of individual particles 1,2 and 3 … n

$\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots \vec{v}_n$ are velocity vectors of individual particles 1,2 and 3 … n

$\vec{r}_{cm}$ and $\vec{V}_{cm}$ are position vector and velocity vector of centre of mass.

Differentiating the velocity expression we will get,

$$M\frac{dv_m}{dt} = m_1\frac{dv_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{dv_3}{dt} + \cdots + m_n\frac{dv_n}{dt} \ldots (4)$$

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n \ldots (5)$$

Where

Where $\vec{a_1}, \vec{a_2}, \vec{a_3}, \ldots \vec{a_n}$ are velocity vectors of individual particles 1,2 and 3

… n and

$\vec{a_{cm}}$ is the acceleration of centre of mass. from Newton's second law of motion.

The force Fi acting on the ith particle is given by,

$$\vec{F_i} = m_i \vec{a_i}$$

The above eq (5) can be written as,

$$M\vec{a_{cm}} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n} = \vec{F_{internal}} + \vec{F_{external}} \ldots \ldots (6)$$

Where F_1, F_2, F_3 and Fn are the forces acting on the individual particles 1, 2 and 3 … n of the system

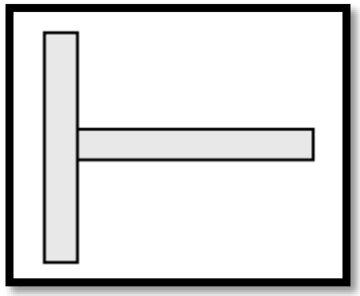
The internal forces are the forces exerted by the particles of the system on each other, however, from newton’s third law, these internal forces occur as pairs of equal magnitude and opposite direction. So their net sum is zero. Then the above eq (6) becomes

$$M\vec{a_{cm}} = \vec{F_{external}} \ldots \ldots (7)$$

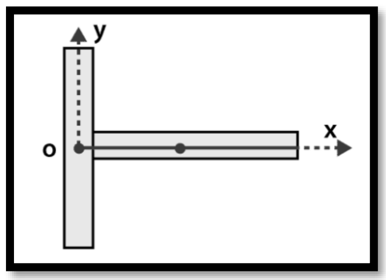
Eq (7) states that the COM of the system of particles behaves like all the mass of the system were concentrated there and the resultant of all the external forces acting on all the particles of the system was applied on COM.

Problems on Center of Mass

Ex. Two identical rods each of mass (m) and length (L) are connected as shown in the figure. Locate the centre of mass of the system.



Sol. This system is symmetrical about x-axis hence we need to find Here we take coordinates of CM of rods.



$M_1 = M_2 = M$

$L_1 = L_2 = L$

Where M_1, M_2 and L_1, L_2 are mass and length of Rod 1 and Rod 2

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{M(0) + M(\frac{L}{2})}{M + M}$$