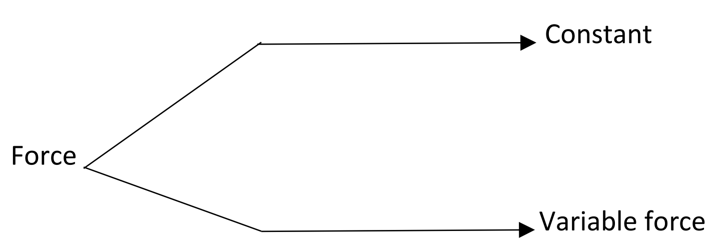
WORK, POWER AND ENERGY

WORK

WORK

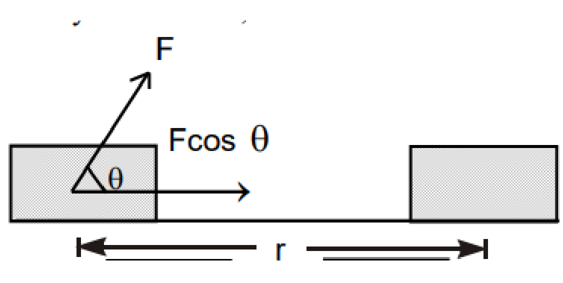
(1) Whenever a force acting on a body displaces it, work is said to be done by the force.

(2) Work done by a force is equal to scalar product of force applied and displacement of the body.



WORK DONE BY A CONSTANT FORCE :

If the direction and magnitude of a force applying on a body is constant, the force is said to be constan



Work done by a constant force,

W = Force × component of displacement along force

= displacement × component of force along displacement

If a $\vec{F}$ force is acting on a body at angle θ to

the horizontal and the displacement $\vec{r}$ is along the horizontal, then the work done will be

$$W = (F\cos\ \theta)r$$

$$= F(r\cos\ \theta)$$

In vector from, $W = \vec{F} \cdot \vec{r}$

If $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$ and $\vec{r} = \hat{i}x + \hat{j}Y + \hat{k}z$

the work done will be, $W = F_y \cdot x + F_y \cdot y + F_z \cdot z$

NOTE

The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force

WORK DONE BYA VARIABLE FORCE

(1) The total work done in displacing body from

P_1 to P_2 is given by,

$$\int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$

$$W = \int_{P_1}^{P_2} \vec{F} \cdot dr$$

If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and

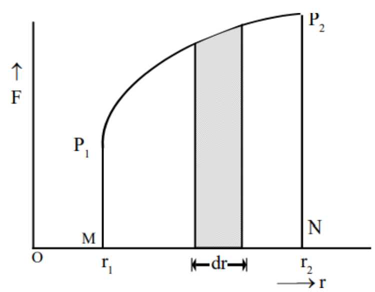
P_2 respectively, the total work done will be -

$$W = \int_{r_r}^{r_2} \vec{F} \cdot dr$$

NOTE

When we consider a block attached to a spring , the force on the block is k times the elongation of the spring , where k is spring constant. As the elongation changes with the motion of the block , therefore the force is variable. This is an example of work done by variable force.

Calculation of work done from force displacement graph :



Suppose a body , whose initial position is r_1, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position r_2. From position r to r + dr or for small displacement dr , the work done will be $\vec{F}.d\vec{r}$ whose value will the area of the shaded strip of width dr. The work done on the body in displacing it from position r_1 to r_2 will be equal to the sum of areas of all the such strips

Thus, total work done,

$$W = \sum^{r_2} dW$$

$$= \sum_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$= \sum_{r}^{r_2} \vec{F} \cdot d\vec{r}$$

$$= \text{ Area of } P_1P_2NM$$

The area between the graph between force and displacement axis is equal to the work done.

NOTE

To calculate the work done by graphical method , for the sake of simplicity , here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cos θ and r.

(1) Work is a scalar quantity

(2) The dimensions of work : $ML^2 T^{-2}$

(3) Unit of work : There are two types of units of work

(1) Absolute unit : joule (in M.K.S.) ,

erg (in C.G.S.) (NOTE : 10^7 erg = 1 joule)

(2) Gravitational unit: Kilogram - metre

(in M.K.S.), Gram-cm

(in C.G.S.)

[NOTE: 1 kilogram metre = 9.8 joule = 10^5 gram cm]

Nature of work done

Although work done is a scalar quantity, yet its value may be positive , negative or even zero.

1) Positive work :

As $W = \vec{F} \cdot \vec{r} = Fr\cos\theta$

When θ is acute ($< 90°$), cos θ is positive. Hence work done is positive

For example:

(i) When a body falls freely under the action of gravity $\theta = 0^{\circ}$, $\cos\theta = +1$, therefore work done by gravity on a body, falling freely is positive.

(ii) When a gas filled cylinder fitted with a movable piston is allowed to expand, work done by the gas is positive. This is because force due to gaseous pressure and displacement of piston are in same direction.

(iii) When a spring is stretched, work done is positive.

2)Negative work:

When θ is obtuse ($>90^{\circ}$), $\cos\theta$ is negative. Hence work done is negative

For example:

(i) When a body is through up , its motion is opposed by gravity. The angle θ between the gravitational force $\vec{F}$ and displacement $\vec{r}$ is 180° . As $\cos\theta = -1$, therefore, work done by gravity is negative.

(ii) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction. Hence work done by frictional force is negative. Note that work done by the applied force is not negative

(iii) When a positive charge is moved closer to another positive charge , work done by electrostatic force of repulsion between the charges is negative.

3) Zero work :

When force $\vec{F}$ or the displacement $\vec{r}$ or both are zero , work done W, will be zero. Again when angle θbetween $\vec{F}$and $\vec{r}$ is 90°, the work done will be zero

For example:

(i) When we fail to move a heavy stone, however hard we may try, work done by us is zero, because $\vec{r} = 0$.

(ii) When a coolie carrying some load on his head moves on horizontal platform , $\theta = 90^{\circ}$. Therefore, work done by the coolie is zero.

(iii) When a body tied to a string is rotated in a circle , the work done by the centripetal force applied along the string is zero. This is because $\theta = 90^{\circ}$

(iv) Tension in the string of simple pendulum is always perpendicular to displacement of the bob. Therefore, work done by tension is always zero.

Ex. A body is acted upon by a force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ The work done by the force in displacing it from (0,0,0) to (0,0,4m) will be –

(1) 12 J (2) 10 J

(3) 8 J (4) 6 J

Sol. (1) Here $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ and

$\vec{d} = (0-0)\hat{i} + (0-0)\hat{j} + (4-0)\hat{k}$

$= 4\hat{k}$

$W = \vec{F} \cdot \vec{d} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k}$

$= 12$ J

Ex. The work done in pulling a body of mass 5 kg along an inclined plane (angle 60°) with coefficient of friction 0.2 through 2 m, will be

(1) 98.08 J (2) 94.08 J

(3) 90.08 J (4) 91.08 J

Sol. (2) The minimum force with a body is to be pulled up along the inclined plane is mg

$(\sin\theta + \mu\cos\theta)$

Work done, $W = \vec{F}\cdot\vec{d}$

$= Fd\cos\theta°$

$= mg(\sin\theta + \mu\cos\theta)\times d$

$= 5\times 9.8(\sin 60° + 0.2\cos 60°)\times 2$

$= 9808I$

Ex. A force $\vec{F} = (7 - 2x + 3x^2)N$ N is applied on a 2 kg mass which displaces it from $x = 0$ to $x = 5$ m. Work done in joule is

(1) 70 (2) 270

(3) 35 (4) 135

Sol. (4)

$$W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\,dx$$

$$= [7x]_0^5 - [\frac{2x^2}{2}]_0^5 + [\frac{3x^3}{3}]_0^5 = 135 \text{ Joule}$$