WORK, POWER AND ENERGY

THE WORK-ENERGY THEOREM FOR A VARIABLE FORCE

Introduction

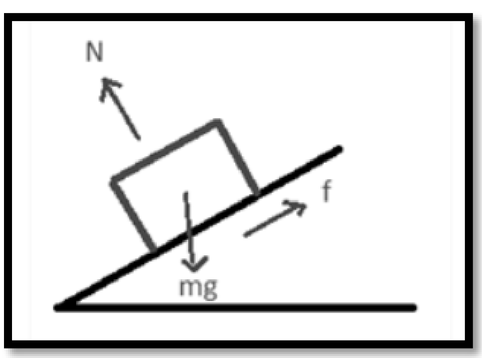
We already discussed in the previous article (link here) that there is some relation between work done and energy. Now we will see the theorem that relates them. According to this theorem, the net work done on a body is equal to the change in kinetic energy of the body. This is known as Work-Energy Theorem. It can be represented as:

$K_f - K_i = W$

Where K_f = Final kinetic energy

- K_i = Initial kinetic energy
- W = net work done

So the above equation follows the law of conservation of energy according to which we can only transfer energy from one form to another. Also here the work done is the work done by all forces acting on the body like gravity, friction, external force etc. For example, consider the following figure,



According to Work energy theorem,

Work done by all the forces = Change in Kinetic Energy

$$W_g + W_N + W_f = K_f - K_i$$

Where

- W_g = work done by gravity
- W_N = work done by a normal force
- W_f = work done by friction
- K_f = final kinetic energy
- K_i = initial kinetic energy

Work done by a constant force

A constant force will produce constant acceleration. Let the acceleration be ‘a’.

From equation of motion,

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

Multiplying both side with mass ‘m’

$$(\mathrm{ma}).\mathrm{s} = \frac{(mv^2 - mu^2)}{2}$$

$$\mathrm{F} \cdot \mathrm{s} = \frac{(mv^2 - mu^2)}{2}$$

Comparing the above equation we get,

Work done by force (F) = F.s

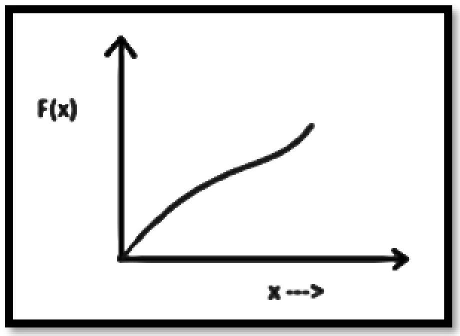
Where ‘s’ is the displacement of the body.

Work done by Non-Uniform Force

Now the equation,

$$W = F.ds$$

This is only valid when force remains constant throughout the displacement. Suppose we have a force represented below,



For these kinds of forces, we can assume that force remains constant for a very small displacement and then integrate that from the initial position to the final position.

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

This is work done by a variable force. A graphical approach to this would be finding the area between F(x) and x from x_i to x_f.