WORK, POWER AND ENERGY

POWER

Power

1. the time rate of doing work is called power
2. power $= \frac{dw}{dt} = \vec{f} \cdot \frac{d\vec{x}}{dt}$

   in translatory motion : $p = \vec{f} \cdot \vec{v}$

   in rotational motion : $P = \vec{\tau} \cdot \vec{\omega}$
3. it is a scalar quantity
4. Unit:

   in mks – j/sec, watt

   in cgs – erg/sec

   (Note: 1kw = 10^3 watt, 1hp = 746 watt)

5. dimension: $[m^1 l^2 t^{-3}]$

   Note:

   power is the rate at which applied force transfers energy

   (1) power $\vec{p} = \frac{w}{\delta t}$ where w work is done in δt

   (2) instantaneous power $p = \frac{dw}{dt}$, it's value may change with time.

Ex. An automobile of mass m accelerates from rest. if the engine supplies constant power p, the velocity at time t is given by –

1. $v = \frac{Pt}{m}$ 2. $v = \frac{2Pt}{m}$

3. $\sqrt{\frac{Pt}{m}}$ 4. $\sqrt{\frac{2Pt}{m}}$

Sol. 4 Given that, power = Fv = P = constant

$$m\frac{dv}{dt}v = P[\text{ as } F = ma = \frac{mdv}{dt}$$

$$\int vdv = \int \frac{P}{m}dt$$

$$\frac{v^2}{2} = \frac{P}{m}t + C_1$$

Now as initially, the body is at rest i.e v = 0 at t = 0 so, $C_1 = 0$

$$v = \sqrt{\frac{2Pt}{m}}$$

Ex. In the example 4, the position (s) at time (t) is given by –

(1) $(\frac{2Pt}{m})t$ (2) $\left(\frac{8P}{9M}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(3) $(\frac{9P}{8m})^{1/2}t^{1/2}$ (4) $(\frac{8P}{9m})^{1/2}t$

Sol. (2) By definition

$$v = \frac{ds}{dt} \text{ or } \frac{ds}{dt} = (\frac{2Pt}{m})^{1/2}$$

[From (1)]

$$\int ds = \int (\frac{2Pt}{m})^{1/2} dt$$

$$s = (\frac{2P}{m})^{1/2} \frac{2}{3} t^{3/2} + C_2$$

Now as $t = 0, s = 0$, so $C_2 = 0$

$$s = (\frac{8P}{9m})^{1/2} t^{3/2}$$

Ex. A particle moving in a straight line is acted by a force, which works at a constant rate and changes its velocity from u to v in passing over a distance x. The time taken will be –

Sol. (3) The force acting on the particle $= \frac{mdv}{dt}$

Power of the force $= \left(\frac{mdv}{dt}\right) v = k(\text{ constant })$

$$\Rightarrow m \frac{v^2}{2} = kt + c$$

at $t = 0, v = u \therefore c = \frac{mu^2}{2}$

Now from (1),

$$m \frac{v^2}{2} = kt + \frac{mu^2}{2}$$

$$\frac{1}{2} m(v^2 - u^2) = kt$$

Again $\frac{mdv}{dt}v = k$

$m \cdot v\frac{dv}{dx}v = k$

$mv^2dv = kdx$

Integrating, $\frac{1}{3}\ m(v^2 - u^3) = kx$

From (2) and (3), $t = \frac{3}{2}(\frac{v^2 - u^2}{v^3 - u^3})(x)$