MOTION IN A PLANE

RESOLUTION OF VECTORS

Resolution of a Vector in a Plane

All Physical quantities like force, momentum, velocity, acceleration are all vector quantities because they have both magnitude and direction. We represent the vector as an arrow-headed line, where the tip of the arrow is the head and the line is the tail.

Let's suppose there are two paths, viz: A and B, where A and B are horizontal and vertical components of a vector, respectively.

$$\sqrt{3^2} + \sqrt{4^2} = \sqrt{9} + \sqrt{16} = \sqrt{25} = 5$$

So, 5 m is displacement.

Now, let's discuss what is the horizontal and vertical component.

Horizontal Component Definition

In science, we define the horizontal component of a force as the part of the force that moves directly in a line parallel to the horizontal axis.

Let's suppose that you kick a football, so now, the force of the kick can be divided into a horizontal component, which is moving the football parallel to the ground, and a vertical component that moves the football at a right angle to the surface/ground.

Vertical Component Definition

We define the vertical component as that part or a component of a vector that lies perpendicular to a horizontal or level plane.

Resolution of a Vector

Resolution of a vector is the splitting of a single vector into two or more vectors in different directions which together produce a similar effect as is produced by a single vector itself. The vectors formed after splitting are called component vectors.

$OB^{\rightarrow} = a_x$ =vector along the x-axis (It is the horizontal component formula)

$OD^{\rightarrow} = a_y$ = vector along the y-axis (It s the vertical component formula)

From here, we obtained the horizontal and vertical components of a vector, which is a vector a

From the triangle law of addition, we can use the formula as:

$OC \rightarrow = OB \rightarrow + OD \rightarrow$

$a^{\rightarrow} = a_x + a_y \ldots (1)$

Here, we can see that OCB is right-angled, so using the formula of the trigonometric function, we get the angular components along the x and y-axis, respectively:

Since

$OB/OC = \text{Cos}$

$OB = OC\cos$

So,

$a_x = a^{\rightarrow}\text{Cos}$

Similarly,

$BC/OC = \text{Sin}$

$a_y = a^{\rightarrow}\text{Sin} \;\ldots$

Now, eq (3) ÷ eq (2), we get the tangent of component, which is given by:

$(a^{\rightarrow}\text{Sin}\ \Theta)/a^{\rightarrow}\text{Cos}\ \Theta() = a_y/a_x$

So,

$\tan = BC/OB = a_y/a_x \ldots (4)$

Rectangular Components of Vectors in Three Dimensions

We define rectangular components of vectors in Three Dimensions in the following manner:

If the coordinates of a point P, i.e., x, y, and z, the vector joining point P to the origin is called the position vector. The position vector of point P is equal to the sum of these coordinates, which is given by:

$x + y + z$

Rectangular Components

Rectangular components of a vector in three dimensions can be better understood by going through the following context:

Let's suppose that vector A→

is presented by the vector OR→

Now, taking O as the origin and construct a rectangular parallelopiped with its three edges along with the three rectangular axes, viz: X, Y, and Z. Here, we can notice that A→

represents the diagonal of the rectangular parallelopiped whose intercepts are the ax, ax, and ax, respectively. We call these intercepts the three rectangular components of A→

Now, using the triangular law of vector addition, we have:

$OR^{\rightarrow} + OT^{\rightarrow} + TR^{\rightarrow}$

Using the parallel law of vector addition, we have:

$OT^{\rightarrow} = OS^{\rightarrow} + OP^{\rightarrow}$

$OR^{\rightarrow} = (OS^{\rightarrow} + OP^{\rightarrow}) + TR^{\rightarrow}$

Here, one must notice that $TR^{\rightarrow} = OQ^{\rightarrow}$. So, rewriting equation (5) in the following manner:

$OR^{\rightarrow} = (OS^{\rightarrow} + OP^{\rightarrow}) + OQ^{\rightarrow}$

Or,

$A^{\rightarrow} = A^{\rightarrow}z + A^{\rightarrow}x + A^{\rightarrow}y = A^{\rightarrow}x + A^{\rightarrow}y + A^{\rightarrow}z$

Therefore,

$A^{\rightarrow} = Ai^{\rightarrow}x + Aj^{\rightarrow}y + Ak^{\rightarrow}Z$

Also,

$OR^2 = OT^2 + TR^2$

$OP^2 + OS^2 + TR^2$

Now,

$A^2 = A^2x + A^2y + A^2z$

Resolution of Rectangular Vectors in Three Dimensions in their Direction Cosines

Now, we can restate equation (6) in the following manner:

$A = \sqrt{A^2x + A^2y + A^2z}$

If α, β, and γ are the angles which the vector A

makes with the X, Y, and Z-axis, respectively, then we have:

$\text{Cos } \alpha = Ax^{\rightarrow}/A^{\rightarrow} \Rightarrow Ax^{\rightarrow} = A^{\rightarrow}\text{Cos } \alpha$

$\text{Cos } \beta = A^{\rightarrow}y/A^{\rightarrow} \Rightarrow A^{\rightarrow}y = A^{\rightarrow}\text{Cos } \beta$

$\text{Cos } Y = Az \rightarrow/A^{\rightarrow} \Rightarrow Az \rightarrow= A^{\rightarrow}\text{Cos } Y$

We must note that Cos α, Cos β, and Cos γ are direction cosines of vectors $Ax^{\rightarrow}$, Ay, and Az,

Now, putting the values of equations (a), (b), and © in the equation (7), we get

$A^2 = A^2\text{Cos}^2\ \alpha + A^2\text{Cos}^2\ \beta + A^2\text{Cos}^2\ y$ ... (8)

So, we get the equation as:

$\text{Cos}^2\,\alpha + \text{Cos}^2\,\beta + \text{Cos}^2\,y = 1$

Here, we conclude that the squares of the direction cosines of three vectors are always constant, i.e., unity.