MOTION IN A PLANE

MULTIPLICATION OF VECTORS BY REAL NUMBERS

Multiplication of vectors is an important tool in many fields of physics, including distance, energies, forces, work done, electric or magnetic fields, and many more applications.

Multiplication of any vector with a real number is known as scalar multiplication. This multiplication alters the magnitude of the vector but does not disturb its direction. However, if the multiplied quantity is negative, this would not be the case, and the direction would be reversed. Newton's second law of motion and the work done are the two most famous examples of applications of scalar multiplication of vectors in physics.

What Happens When We Multiply Vectors By Real Numbers

When we multiply a vector by a real number, the magnitude of the vector changes, however, there is no effect on the direction of the vector.

For example

$|p\cdot\bar{A}| = p|\bar{A}|$ if $p>0$.

If $p<0$, then the magnitude and direction of the vector both change.

Multiplication of Vectors

Multiplication of vectors has an application in various fields of physics like distance, energies, forces, work done, electric or magnetic field, and many more. Vectors can be multiplied in two ways, namely scalar multiplication and vector multiplication. We will discuss scalar multiplication, that is, the multiplication of a vector with a real number.

Scalar Multiplication

Multiplication of any vector with a real number is referred to as scalar multiplication. Whenever we multiply a scalar quantity with any vector quantity, the magnitude of a

vector changes. However, the change of its direction depends on the scalar quantity that we multiply it with. If the scalar quantity is positive, the direction remains the same, and if the scalar quantity is negative, the direction becomes the opposite of the original direction. Also, if we multiply the vector with zero, it becomes a null vector (zero vector).

The scalar product of two vectors is the dot product of their components. The scalar product of two vectors can be used to transform a vector into a scalar that represents its value. In other words, the scalar product of two vectors is a scalar value that represents the sum of the elements of each vector.

Scalar Product formula = $A \bullet B = |A|\,|B| \cos\theta$

Properties of Scalar Multiplication

$a \cdot b$ is a scalar quantity

If the dot product of two vectors is zero, it means that they are perpendicular to each other. This is because

$A \bullet B = |A|\,|B| \cos\theta = 0$

$\cos\theta = 0$ means $\theta = 90°$

If the dot product of two quantities equals the product of their magnitudes, it means the two vectors are parallel. This is because $\cos\theta = 1$ means $\theta = 0°$

Similarly, if the dot product of two vectors equals the negative of the product of their magnitudes, it implies that they are antiparallel. This is because, for $\cos\theta = -1$, the value of θ will be 180°.

While solving scalar products,

$i \cdot i = j \cdot j = k \cdot k = 1$ and $i \cdot j = i \cdot k = j \cdot k = 0$

because of the same reasons as mentioned above.

$\cos\theta = a.b / |a||b|$

Dot products (Scalar products) are commutative in nature, that is, $a \cdot b = b \cdot a$

They also follow the distributive property that is $a \cdot (b+c) = a \cdot b + a \cdot c$

Newton's Second Law of Motion

Newton's second law of motion is the best example for the multiplication of vectors by a real number. It states that the force is directly proportional to the acceleration of the motion. When we address the proportionality, we multiply the equation with the constant value mass. The equation comes out to be

$F = ma$

Here, force and acceleration both are vector quantities, whereas m is a scalar quantity. Force is obtained by the multiplication of a vector quantity by a real number. We can see that this multiplication of acceleration only changes its magnitude to give the magnitude of the force, and does not change the direction of the force. The force is applied in the direction of acceleration itself. Mass (m) is always a positive quantity. Hence, there is no scope for it to be less than zero, and hence, the direction of the force will not be changed.

Conclusion

Vectors can be multiplied in two ways, namely scalar multiplication and vector multiplication. Multiplication of vectors has applications in various fields of physics like distance, energies, forces, work done, electric or magnetic field, and many more, whenever we multiply a scalar quantity with any vector quantity, the magnitude of a vector changes. If the scalar quantity is positive, the direction remains the same, and if the scalar quantity is negative, the direction becomes the opposite of the original direction. Newton's second law of motion states that the force is directly proportional to the acceleration of the motion, and is thus a good example of scalar multiplication.