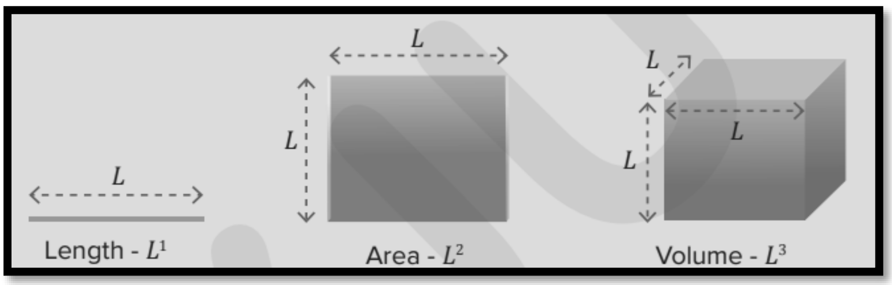
UNITS AND MEASUREMENTS

DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

Dimensions

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.



Symbols for dimensions of fundamental quantities

Following are the symbols used for writing dimensions of fundamental quantities which are used combinedly to express derived quantities as well.

Fundamental Quantities	Dimensional Symbol
Mass	M
Length	L
Time	T
Electric current	A
Temperature	K
Luminous intensity	cd
Mole	mol

Dimension Formulae of Certain Quantities

Physical Quantity	Mathematical Formula	SI Unit	Dimension Formula
Area	$l \times b$	m^2	$[L^2]$
Volume	$l \times b \times h$	m^3	$[L^3]$
Velocity	$\frac{s}{t}$	ms^{-1}	$[LT^{-1}]$
Momentum	mv	$kg\ ms^{-1}$	$[MLT^{-1}]$
Force	ma	$kg\ ms^{-2}$ (N)	$[MLT^{-2}]$
Work done	Fs	$kg\ m^2s^{-2}$ (J)	$[ML^2\ T^{-2}]$
Displacement	l	m	$[L]$
Acceleration	$\frac{v}{t}$	ms^{-2}	$[LT^{-2}]$
Kinetic energy	$\frac{1}{2}mv^2$	$kg\ m^2s^{-2}$ (J)	$[ML^2\ T^{-2}]$
Pressure	$\frac{F}{A}$	$kg\ m^{-1}s^{-2}$ (Nm^{-2})	$[ML^{-1}T^{-2}]$
Torque	Fr	$kg\ m^2s^{-2}$	$[ML^2\ T^{-2}]$
Moment of inertia	mR^2	$kg\ m^2$	$[ML^2]$
Angular displacement	θ	radian	Dimensionless
Angular velocity	$\frac{\theta}{t}$	$rad\ s^{-1}$	$[T^{-1}]$
Stress	$\frac{F}{A}$	$kg\ m^{-1}s^{-2}$	$[ML^{-1}T^{-2}]$
Strain	$\frac{\Delta l}{l}$	Unitless	Dimensionless
Charge	$I \times t$	coulomb	$[AT]$

Dimensional Correctness of Equation

Every physical equation should be dimensionally balanced.

To check the dimensional correctness of a given physical equation, we use the principle of homogeneity.

Dimensionally correct equation may or may not be physically correct

Principle of homogeneity

This principle states that the dimensions of all the terms in a physical expression should be the same. Example:

$$s = ut + \frac{1}{2}at^2$$

$$[L] = [LT^{-1}][T] + [LT^{-2}][T^2]$$

$$[L] = [L] + [L]$$

Ex. Show that the expression of the time period T of a simple pendulum of length l given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

is dimensionally correct

Sol. Dimension of LHS = [T

Dimensional Formula

The dimensional formula of any physical quantity is the formula that tells which of the fundamental units have been used for the measurement of that physical quantity.

dimensional formula is written for a physical quantity

1. The formula of the physical quantity must be written. The quantity must be on the left-hand side of the equation.
2. All the quantities on the right-hand side of the formula must be written in terms of fundamental quantities like mass, length and time.
3. Replace mass, length and time with M, L and T.
4. Write the powers of the terms.

Characteristics of Dimensions

1. Dimensions do not depend on the system of units.
2. Quantities with similar dimensions can be added or subtracted from each other.
3. Dimensions can be obtained from the units of the physical quantities and vice versa.
4. Two different quantities can have the same dimension.
5. When two dimensions are multiplied or divided it will form the dimension of the third quantity.

Uses of Dimension

1. It is used in conversion of units.
2. Dimensional correctness of equation: Dimension of both LHS and RHS in an equation should be the same.
3. It establishes a relation between physical quantities.

Conversion of units

Dimension is used to convert the numerical value of a physical quantity from one system of units into the other system.

Example:

Velocity from CGS system (100 cm s^{-1}) to SI system (1 ms^{-1})

The product of the numerical value (n) and its corresponding unit (u) is always constant.

$n[u] = \text{Constant}$

$n_1 [u_1] = n_2 [u_2]$

$100 \text{ cm } s^{-1} = n_2 \times ms^{-1}$

$100 \text{ cm } s^{-1} = n_2 \times 100 \text{ cm } s^{-1}$

$n_2 = 1$