MECHANICAL PROPERTIES OF FLUID

BERNOULLI'S PRINCIPLE

Bernoulli's Principle

Bernoulli's principle states that

The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.

Bernoulli's principle can be derived from the principle of conservation of energy.

Bernoulli's Principle Formula

Bernoulli's equation formula is a relation between pressure, kinetic energy, and gravitational potential energy of a fluid in a container.

The formula for Bernoulli's principle is given as follows:

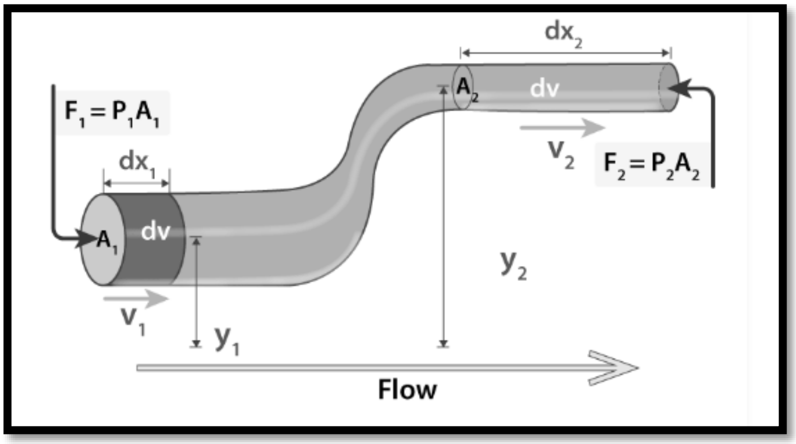
$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{ constant}$$

Where p is the pressure exerted by the fluid, v is the velocity of the fluid, ρ is the density of the fluid and h is the height of the container.

Bernoulli's equation gives great insight into the balance between pressure, velocity and elevation.

Bernoulli's Equation Derivation

Consider a pipe with varying diameter and height through which an incompressible fluid is flowing. The relationship between the areas of cross-sections A, the flow speed v, height from the ground y, and pressure p at two different points 1 and 2 are given in the figure below.



Assumptions:

The density of the incompressible fluid remains constant at both points.

The energy of the fluid is conserved as there are no viscous forces in the fluid.

Therefore, the work done on the fluid is given as:

$dW = F_1dx_1 - F_2dx_2$

$dW = p_1A_1dx_1 - p_2A_2dx_2$

$dW = p_1dv - p_2dv = (p_1 - p_2)dv$

We know that the work done on the fluid was due to the conservation of change in gravitational potential energy and change in kinetic energy. The change in kinetic energy of the fluid is given as:

$$dK = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}\rho dv(v_2^2 - v_1^2)$$

The change in potential energy is given as:

$dU = m_2gy_2 – m_1gy_1 = \rho dvg(y_2 – y_1)$

Therefore, the energy equation is given as:

$dW = dK + dU$

$$(p_1 - p_2)dv = \frac{1}{2}\rho dv(v_2^2 - v_1^2) + \rho dvg(y_2 - y_1)$$

$$p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

Rearranging the above equation, we get

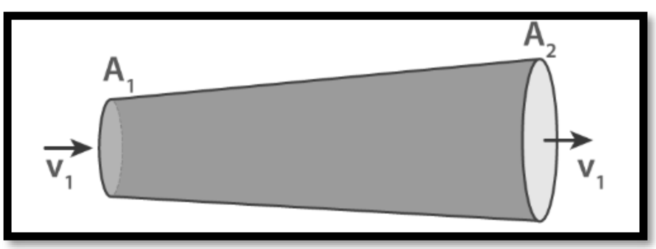
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2$$

This is Bernoulli's equation.

Principle of Continuity

According to the principle of continuity

If the fluid is in streamline flow and is in-compressible then we can say that mass of fluid passing through different cross sections are equal.



From the above situation, we can say the mass of liquid inside the container remains the same.

The rate of mass entering = Rate of mass leaving

The rate of mass entering = $\rho A_1 V_1 \Delta t$—– (1)

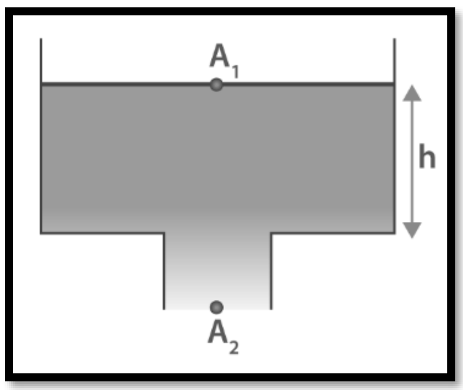
The rate of mass entering = $\rho A_2 V_2 \Delta t$—– (2)

Using the above equations,

$\rho A_1 V_1 = \rho A_2 V_2$

This equation is known as the Principle of Continuity.

Suppose we need to calculate the speed of efflux for the following setup.



Using Bernoulli's equation at point 1 and point 2,

$$p + \frac{1}{2}\rho v_1^2 + \rho gh = p_0 + \frac{1}{2}\rho v_2^2$$

$$v_2^2 = v_1^2 + 2p\rho - 2p_0\rho + 2gh$$

Generally, A_2 is much smaller than A_1; in this case, v_1^2 is very much smaller than v_2^2 and can be neglected. We then find,

$$v_2^2 = 2\frac{p - p_0}{\rho} + 2gh$$

Assuming $A_2 << A_1$,

We get,

$$v_2 = \sqrt{2gh}$$

Hence, the velocity of efflux is

$$\sqrt{2gh}$$

Bernoulli's Principal Use

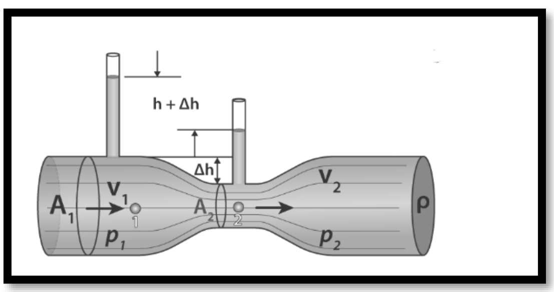
Bernoulli's principle is used for studying the unsteady potential flow which is used in the theory of ocean surface waves and acoustics. It is also used for approximation of parameters like pressure and speed of the fluid.

The other applications of Bernoulli's principle are:

Venturi meter:

It is a device that is based on Bernoulli's theorem and is used for measuring the rate of flow of liquid through the pipes. Using Bernoulli's theorem, Venturi meter formula is given as:

$$V = A_1 A_2 \sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$



Working of an aeroplane: The shape of the wings is such that the air passes at a higher speed over the upper surface than the lower surface. The difference in airspeed is calculated using Bernoulli's principle to create a pressure difference.

When we are standing at a railway station and a train comes we tend to fall towards the train. This can be explained using Bernoulli's principle as the train goes past, the velocity of air between the train and us increases. Hence, from the equation, we can say that the pressure decreases. So the pressure from behind pushes us towards the train. This is based on Bernoulli's effect.

Relation between Conservation of Energy and Bernoulli's Equation

Conservation of energy is applied to the fluid flow to produce Bernoulli's equation. The net work done results from a change in a fluid's kinetic energy and gravitational potential energy. Bernoulli's equation can be modified depending on the form of energy involved. Other forms of energy include the dissipation of thermal energy due to fluid viscosity.

Bernoulli's Equation at Constant Depth

When the fluid moves at a constant depth that is when $h_1 = h_2$, then Bernoulli's equation is given as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Bernoulli's Equation for Static Fluids

When the fluid is static, then $v_1 = v_2 = 0$, then Bernoulli's equation is given as:

When $v_1 = v_2 = 0$ $\quad P_1 + \rho g h_1 = P_2 + \rho g h_2$

When $h_2 = 0$ $\quad P_2 = P_1 + \rho g h_1$