BINOMIAL THEOREM

INTRODUCTION OF BINOMIAL THEOREM

BINOMIAL EXPRESSION

Any algebraic expression which contains two dissimilar terms is called Binomial expression.

For example : x-y, xy + $\frac{1}{x}$, $\frac{1}{z}$ -1, $\frac{1}{(x-y)^{\frac{1}{3}}}$ +3 etc.

Terminology Used in Binomial Theorem

Factorial notation : $\underline{|n}$ or n! is pronounced as factorial n and is defined as

$$n! = \begin{cases} n(n-1)(n-2)......3.2.1 & \text{if } n \in N \\ 1 & \text{if } n = 0 \end{cases}$$

v $n! = n \cdot (n-1)!$; $n \in N$

BINOMIAL THEOREM

The formula by which any positive integral power of a Binomial expression can be expanded in the form of a series is known as **BINOMIAL THEOREM.**

If x, y R and nN, then :

$(x + y)^n = {}^nC_0x^n + {}^nC_1x^{n-1} y + {}^nC_2x^{n-2} y^2 + ... + {}^nC_rx^{n-r} y^r + ... + {}^nC_ny^n =$

This theorem can be proved by induction.

Note:

(a) The number of terms in the expansion is (n+1) i.e. one more than the index.

(b) The sum of the indices of x & y in each term is n.

(c) The Binomial coefficients of the terms (${}^nC_0$, ${}^nC_1$.....) equidistant from the beginning and the end are equal.

i.e. ${}^nC_r = {}^nC_{n-r}$

(d) Symbol ${}^nC_r$ can also be denoted by $\binom{n}{r}$, **C (n,r)** or .

- The coefficient of x^r in $(1+x)^n = {}^nC_r$ & that in $(1-x)^n = (-1)^r \cdot {}^nC_r$

Some Important Expansions :

(i) $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots\ldots + {}^nC_n x^n.$

(ii) $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 + \ldots\ldots + (-1)^n . {}^nC_n x^n.$

Ex. Expand the following Binomials :

(i) $(x-3)^5$ (ii) $\left(1-\frac{3x^2}{2}\right)^4$

Sol.

(i) $(x-3)^5 = {}^5C_0x^5 + {}^5C_1x^4(-3)^1 + {}^5C_2\, x^3(-3)^2 + {}^5C_3\, x^2(-3)^3 + {}^5C_4\, x(-3)^4 + {}^5C_5\, (-3)^5$

$= x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

(ii) $\left(1-\frac{3x^2}{2}\right) = C_0^4 + C_1^4\left(\frac{-3x^2}{2}\right) + C_2^4\left(\frac{-3x^2}{2}\right)^2 + C_3^4\left(\frac{-3x^2}{2}\right)^3 + C_4^4\left(\frac{-3x^2}{2}\right)^4$

$= 1 - 6x^2 + \frac{27}{2}x^4 - \frac{27}{2}x^6 + \frac{81}{16}x^8$

Ex. Find the value of $\frac{(18^3+7^3+3.18.7.25)}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$

Sol. The numerator is of the form $a^3 + b^3 + 3ab(a+b) = (a+b)^3$

Where, $a = 18$ and $b = 7$ $\therefore$ $N^r = (18+7)^3 = (25)^3$

Denominator can be written as

$\therefore \frac{Nr}{Dr} = \frac{(25)^3}{(25)^3} = 1$