TRIGONOMETRIC FUNCTIONS

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TRIGONOMETRIC FUNCTIONS (T-RATIOS)

Trigonometric Functions

1. $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{MP}{OP}$

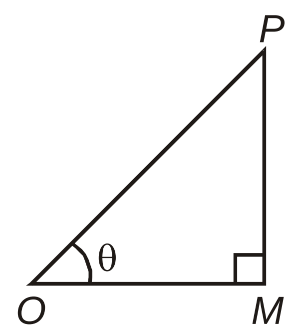
2. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OM}{OP}$

3. $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{MP}{OM}$

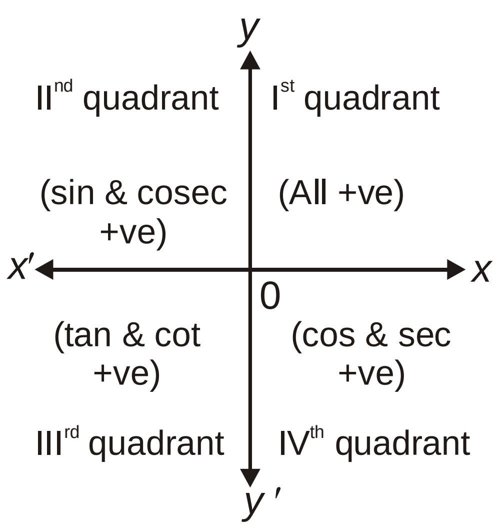
4. $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{MP}$

5. $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OP}{OM}$

6. $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OP}{MP}$



Signs of T-Ratios



Domain and Range of Trigonometric Functions

Function	Domain	Range
$\sin\theta$	R	$[-1,1]$
$\cos\theta$	R	$[-1,1]$
$\tan\theta$	$R \sim \left\{(2n+1)\frac{\pi}{2} : n \in I\right\}$	R
$\cot\theta$	$R \sim \{n\pi : n \in I\}$	R
$\sec\theta$	$R \sim \left\{(2n+1)\frac{\pi}{2} : n \in I\right\}$	$(-\infty,-1] \cup [1,\infty)$
$\text{cosec}\,\theta$	$R \sim \{n\pi : n \in I\}$	$(-\infty,-1] \cup [1,\infty)$

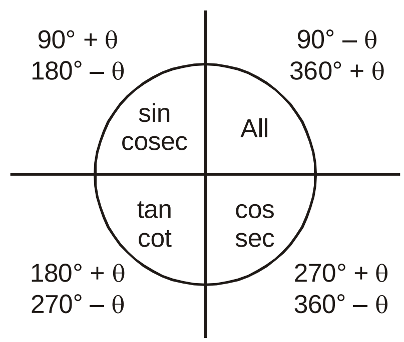
Allied Angle

If q is any angle then, –q, 90° ± q, 180° ± q, 270° ± q, 360° ± q etc. are called as allied angles of q.

1. To find the sign (+ or –)

   Use the original ratio to find '+ or –' sign (to be affixed) making use of the quadrant rule.

   Thus, ratios shown inside the circle are positive in the corresponding quadrant while other ratios are negative there



2. To find the final ratio

(a) If p, 2p etc. are present, then there is no change ;
i.e., sin remains sin ; cos remains cos etc.

(b) If $\frac{\pi}{2}, \frac{3\pi}{2}$ are present, then, there is a change as given below :

$$\sin \rightleftharpoons \cos \quad \tan \rightleftharpoons \cot \quad cosec \rightleftharpoons \sec$$

	$-\theta$	$90° - \theta$	$90° + \theta$	$180° - \theta$	$180° + \theta$	$270° - \theta$	$270° + \theta$	$360° - \theta$	$360° + \theta$
$\sin\theta$	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
$\tan\theta$	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$

Ex.1 Find the values of the cos (–1710°)

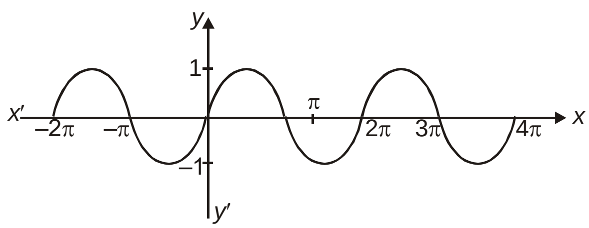
Sol. $\cos(-1710°) = \cos 1710°$ [cos(–q) = cosq]

$= \cos(5 \times 360° - 90°)$

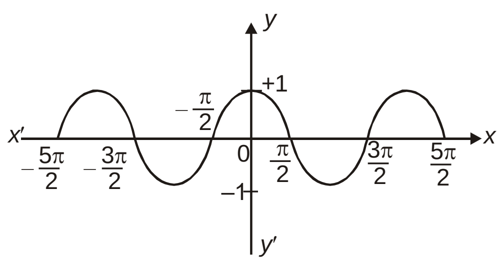
$= \cos(-90°)$

$= \cos 90° = 0$

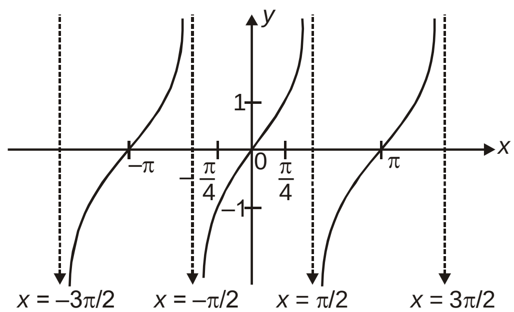
Graphs of standard T-Functions



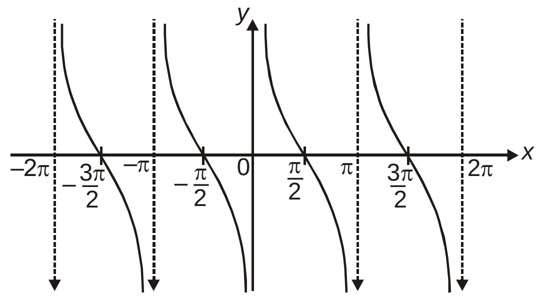
y = sinx



y = cosx



y = tanx



y = cotx

RIGONOMETRIC IDENTITIES

Fundamental Identities

(i) $\sin^2 q + \cos^2 q = 1 \Rightarrow \sin^2 q = 1 - \cos^2 q \Rightarrow \cos^2 q = 1 - \sin^2 q$

(ii) $1 + \tan^2 q = \sec^2 q \Rightarrow \sec^2 q - \tan^2 q = 1$

(iii) $1 + \cot^2 q = \mathrm{cosec}^2 q \Rightarrow \mathrm{cosec}^2 q - \cot^2 q = 1$

Also note the range within which different trigonometric function lie

(1) $-1 \le \sin q \le 1$; $|\sin q| \le 1$

(2) $-1 \le \cos\theta \le 1$; $|\cos\theta| \le 1$

(3) $0 \le \sin^2\theta \le 1$; $0 \le \cos^2\theta \le 1$

(4) $\mathrm{cosec}\,\theta \le -1$ or $\mathrm{cosec}\,\theta \ge 1$

(5) $\sec\theta \le -1$ or $\sec\theta \ge 1$

(6) $0 < \cos A < \frac{\sin A}{A} < \frac{1}{\cos A}; 0 < A < \frac{\pi}{2}$

(7) If $\theta <<<$ then $\sin\theta \simeq \theta$

Note : Each trigonometric ratio can be expressed in terms of all other t-ratios e.g.

$$\sin\theta = \frac{\pm 1}{\sqrt{1+\cot^2\theta}}; \cos\theta - \frac{\pm\cot\theta}{\sqrt{1+\cot^2\theta}}$$

$$\tan\theta = \frac{1}{\cot\theta}; \sec\theta = \frac{\pm\sqrt{1+\cot^2\theta}}{\cot\theta}$$

$\operatorname{cosec}\theta = \pm\sqrt{1+\cot^2\theta}$

Addition and Subtraction Formulae (Compound Angle)

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
3. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
4. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
5. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
6. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
7. $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$
8. $\cot(A - B) = \dfrac{\cot A \cot B + 1}{costB - \cos A}$
9. $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$
10. $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$
11. $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$

    or

    $= \cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \tan B \tan C)$
12. $\cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C$

    or

    $= \cos A \cos B \cos C (1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$
13. $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$
14. $\tan\left(\dfrac{\pi}{4} + A\right) = \dfrac{1 + \tan A}{1 - \tan A}$
15. $\tan\left(\dfrac{\pi}{4} - A\right) = \dfrac{1 - \tan A}{1 + \tan A}$

Two special series :

1. sin(a) + sin(a + b) + sin(a + 2b) + ... sin(a+(n – 1)b)

$$= \frac{\sin\left[\alpha + (n-1)\left(\frac{\beta}{2}\right)\right] \cdot \sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

2. cosa + cos(a + b) + cos(a + 2b) + ...+ cos(a + (n – 1)b)

$$= \frac{\cos\left[\alpha + (n-1)\left(\frac{\beta}{2}\right)\right] \cdot \sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$