RELATIONS AND FUNCTIONS

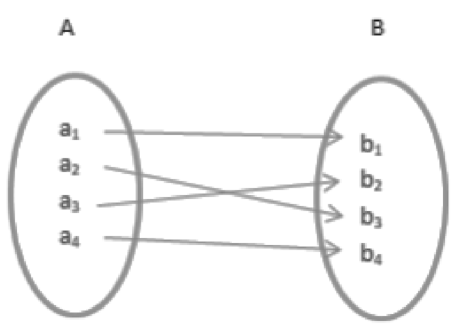
TYPES OF FUNCTION AND ALGEBRA OF REAL FUN

FUNCTIONS

We can define a function as a special relation which maps each element of set A with one and only one element of set B. Both the sets A and B must be non-empty. A function defines a particular output for a particular input. Hence, f: A → B is a function such that for a ∈ A there is a unique element b ∈ B such that (a, b) ∈ f

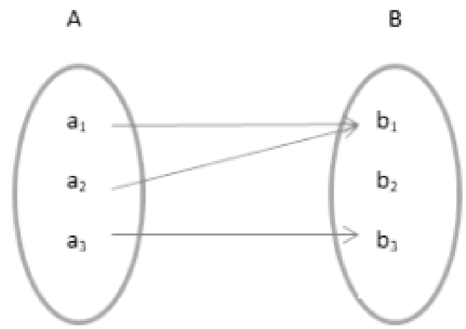
One to One Function

A function f: A → B is One to One if for each element of A there is a distinct element of B. It is also known as Injective. Consider if $a_1 \in A$ and $a_2 \in B$, f is defined as f: A → B such that f (a_1) = f (a_2)



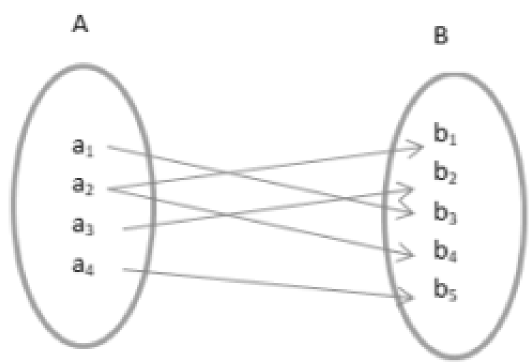
Many to One Function

It is a function which maps two or more elements of A to the same element of set B. Two or more elements of A have the same image in B.



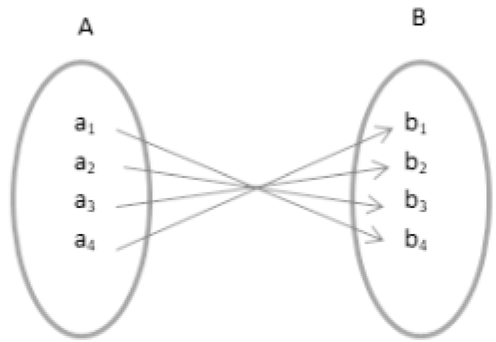
Onto Function

If there exists a function for which every element of set B there is (are) pre-image(s) in set A, it is Onto Function. Onto is also referred as Surjective Function.



One – One and Onto Function

A function, f is One – One and Onto or Bijective if the function f is both One to One and Onto function. In other words, the function f associates each element of A with a distinct element of B and every element of B has a pre-image in A.

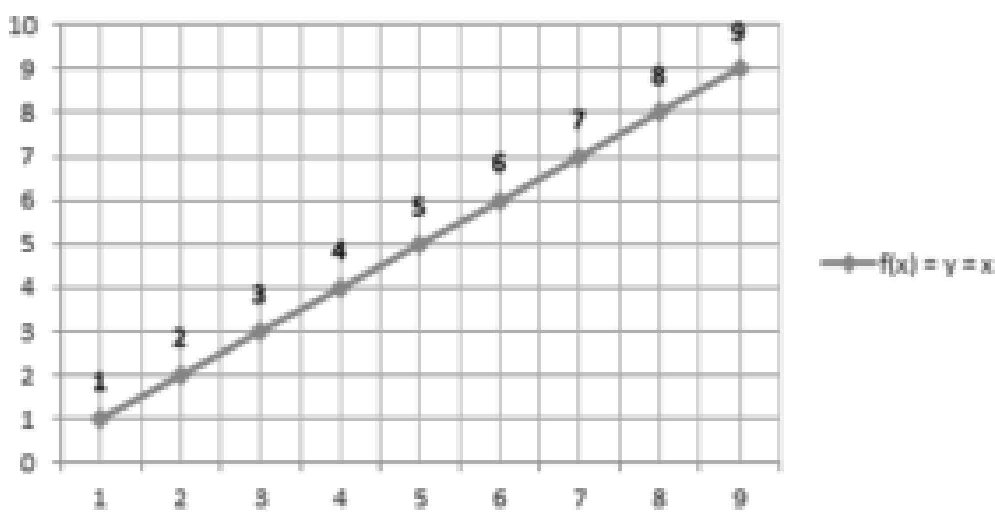


Other Types of Functions

A function is uniquely represented by its graph which is nothing but a set of all pairs of x and f(x) as coordinates. Let us get ready to know more about the types of functions and their graphs.

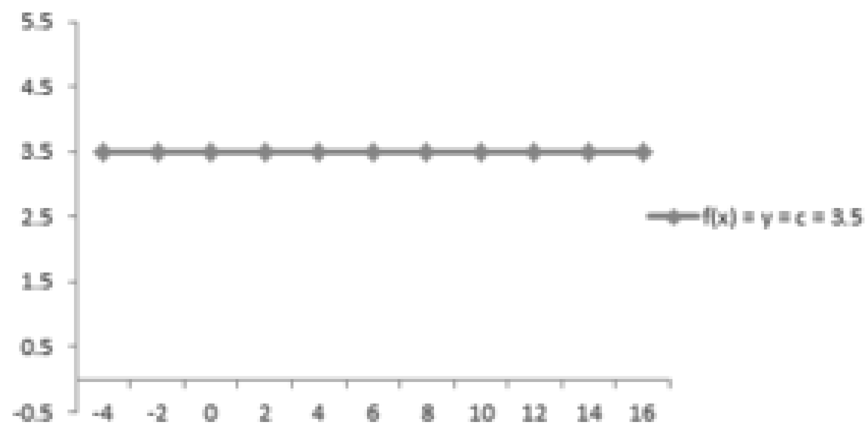
Identity Function

Let R be the set of real numbers. If the function f: R→R is defined as $f(x) = y = x$, for $x \in R$, then the function is known as Identity function. The domain and the range being R. The graph is always a straight line and passes through the origin.



Constant Function

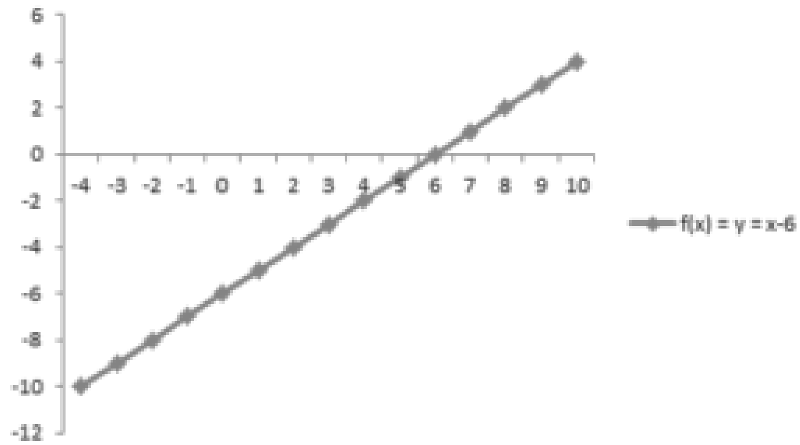
If the function f: R→R is defined as $f(x) = y = c$, for $x \in R$ and c is a constant in R, then such function is known as Constant function. The domain of the function f is R and its range is a constant, c. Plotting a graph, we find a straight line parallel to the x-axis.



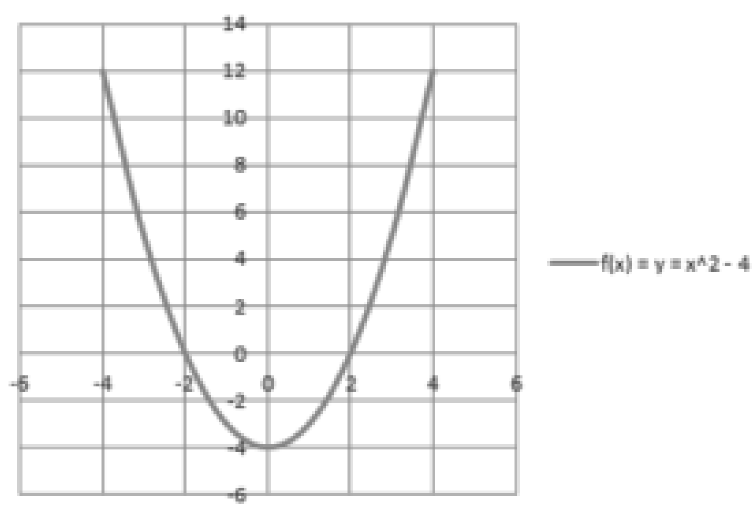
Polynomial Function

A polynomial function is defined by $y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots, {}_n \in R$. The highest power in the expression is the degree of the polynomial function. Polynomial functions are further classified based on their degrees:

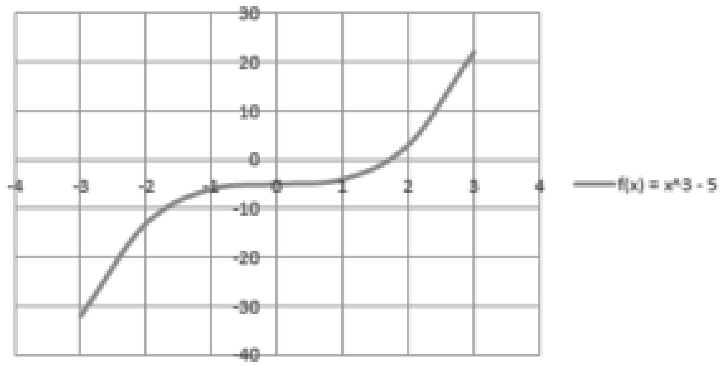
- Constant Function: If the degree is zero, the polynomial function is a constant function (explained above).
- Linear Function: The polynomial function with degree one. Such as $y = x + 1$ or $y = x$ or $y = 2x – 5$ etc. Taking into consideration, $y = x – 6$. The domain and the range are R. The graph is always a straight line.



Quadratic Function: If the degree of the polynomial function is two, then it is a quadratic function. It is expressed as $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constant & x is a variable. The domain and the range are R. The graphical representation of a quadratic function say, $f(x) = x^2 - 4$ is

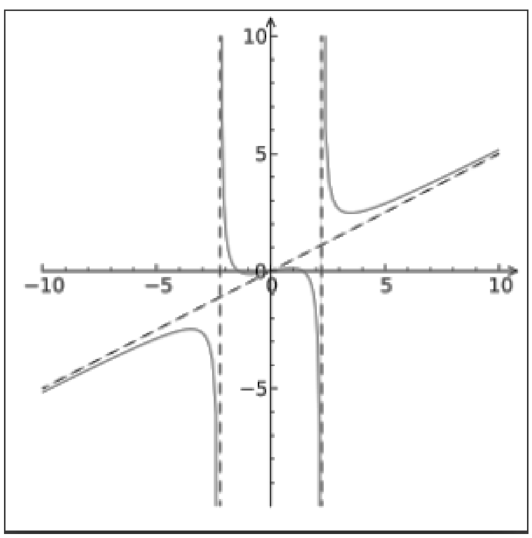


- Cubic Function: A cubic polynomial function is a polynomial of degree three and can be denoted by $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, and d are constant & x is a variable. Graph for $f(x) = y = x^3 - 5$. The domain and the range are R.



Rational Function

A rational function is any function which can be represented by a rational fraction say, f(x)/g(x) in which numerator, f(x) and denominator, g(x) are polynomial functions of x, where $g(x) \neq 0$. Let a function f: R → R is defined say, $f(x) = 1/(x + 2.5)$. The domain and the range are R. The Graphical representation shows asymptotes, the curves which seem to touch the axes-lines.

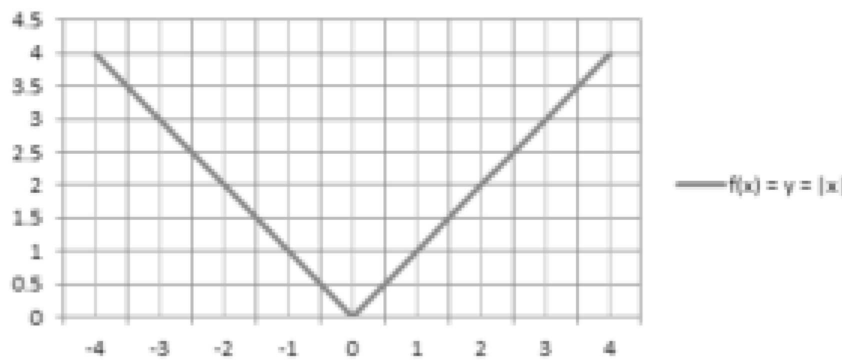


Modulus Function

The absolute value of any number, c is represented in the form of |c|. If any function f: R→ R is defined by f(x) = |x|, it is known as Modulus Function. For each non-negative value of x,

f(x) = x and for each negative value of x, f(x) = -x, i.e., f(x) = {x, if x ≥ 0; – x, if x < 0.

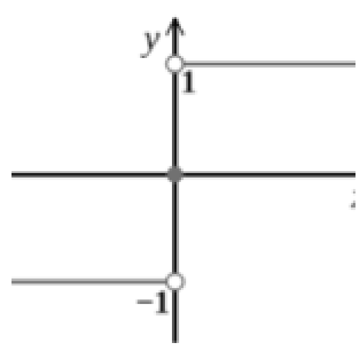
Its graph is given as, where the domain and the range are R.



Signum Function

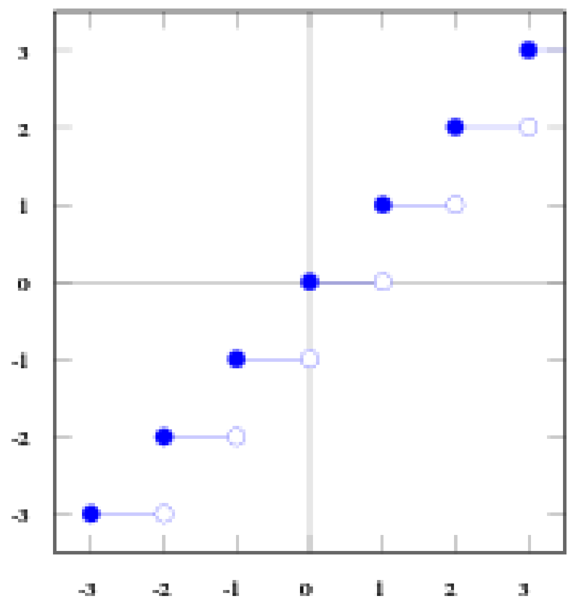
A function f: R→ R defined by f(x) = { 1, if x > 0; 0, if x = 0; -1, if x < 0

Signum or the sign function extracts the sign of the real number and is also known as step function.

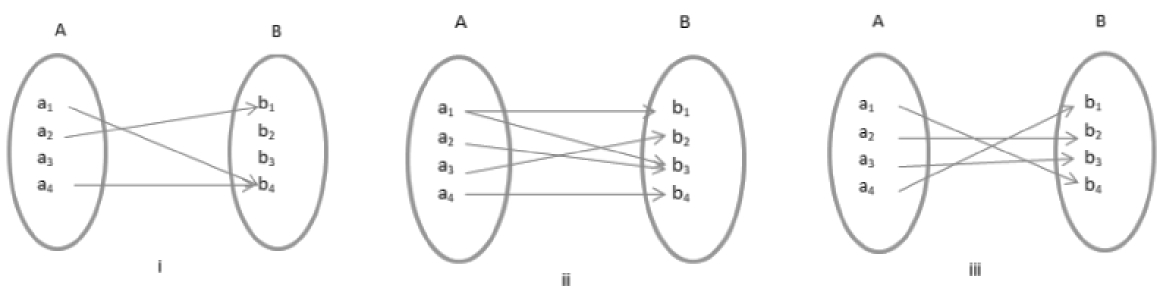


Greatest Integer Function

If a function f: R→ R is defined by f(x) = [x], x ∈ X. It round-off to the real number to the integer less than the number. Suppose, the given interval is in the form of (k, k+1), the value of greatest integer function is k which is an integer. For example: [-21] = 21, [5.12] = 5. The graphical representation is



Ex.1 Which of the following is a function?



Sol. Figure (iii) is an example of a function. Since the given function maps every element of A with that of B. In figure (ii), the given function maps one element of A with two elements of B (one to many). Figure (i) is a violation of the definition of the function. The given function does not map every element of A.

Ex.2 What is meant by function and what are its types?

Sol. A function refers to a special relation which maps each element of one set with only one element belonging to another set. The various types of functions are as follows:

- Many to one function
- One to one function
- Onto function
- One and onto function
- Constant function
- Identity function
- Quadratic function
- Polynomial function
- Modulus function
- Rational function
- Signum function
- Greatest integer function

Algebra of Real Functions

we will get to know about addition, subtraction, multiplication, and division of real mathematical functions with another.

Addition of Two Real Functions

Let f and g be two real valued functions such that f: $X \to R$ and g: $X \to R$ where $X \subset R$. The addition of these two functions $(f + g) : X \to R$ is defined by:

$(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

Subtraction of One Real Function from the Other

Let f: $X \to R$ and g: $X \to R$ be two real functions where $X \subset R$. The subtraction of these two functions $(f - g): X \to R$ is defined by:

$(f - g)(x) = f(x) - g(x)$, for all $x \in X$.

Multiplication by a Scalar

Let f: $X \to R$ be a real-valued function and γ be any scalar (real number). Then the product of a real function by a scalar $\gamma f: X \to R$ is given by:

$(\gamma f)(x) = \gamma f(x)$, for all $x \in X$.

Multiplication of Two Real Functions

The product of two real functions say, f and g such that f: $X \to R$ and g: $X \to R$, is given by

$(fg)(x) = f(x) g(x)$, for all $x \in X$.

Division of Two Real Functions

Let f and g be two real-valued functions such that f: X→R and g: X→R where X ⊂ R. The quotient of these two functions (f / g): X→R is defined by:

(f / g) (x) = f(x) / g(x), for all x ∈ X.

Note: It is also called pointwise multiplication.

Ex.3 Let $f(x) = x^3$ and g(x) = 3x + 1 and a scalar, γ= 6. Find

1. (f + g) (x)
2. (f – g) (x)
3. (γf) (x)
4. (γg) (x)
5. (fg) (x)
6. (f / g) (x)

Sol. We have

1. (f + g) (x) = f(x) + g(x) = $x^3 + 3x + 1$.
2. (f – g) (x) = f(x) – g(x) = $x^3 - (3x + 1) = x^3 - 3x - 1$.
3. (γf) (x) = γ f(x) = $6x^3$
4. (γg) (x) = γ g(x) = 6 (3x + 1) = 18x + 6.
5. (fg) (x) = f(x) g(x) = $x^3 (3x +1) = 3x^4 + x^3$.
6. (f / g) (x) = f(x) / g(x) = $x^3 / (3x +1)$, provided x ≠ – 1/3.