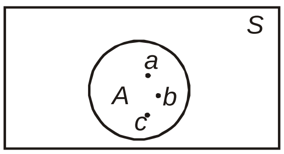
SETS

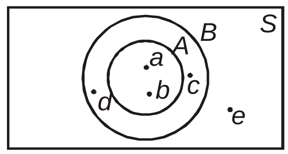
VENN DIAGRAM AND OPERATION ON SETS

VENN DIAGRAMS

Introduced by Euler (a Swiss mathematician) and named after John Venn. It is a pictorial representation of sets in which a set is represented by a circle or a closed geometrical figure inside universal set which is shown by a rectangle. Each element of a set is represented by a point within the circle representing that set.



$A = \{a, b, c\}$

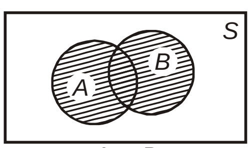


$A \subset B$; $A = \{a, b\}$; $B = \{a, b, c, d\}$; $e \notin A$; $e \notin B$

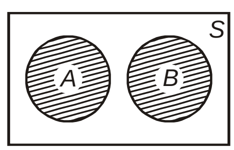
Various Operations on Sets

Union of Sets : $A \cup B$ (read as 'A union B' or 'A cup B' or 'A join B') is a set consisting of all the elements which are either in A or in B or in both.

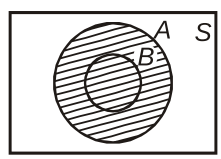
$A \cup B = \{x : x \in A, x \in B\}$



$A \cup B$



$A \cup B$



$A \cup B$

$x \in A \cup B \Leftrightarrow x \in A \vee x \in B$

$x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$ 'v'denotes 'or'

Example:

$Q \rightarrow$ Set of all rational numbers

$Q' \rightarrow$ Set of all irrational numbers

$R = Q \cup Q'$

$A_1 \cup A_2 \cup A_3$ .... $\cup A_n = \bigcup_{i=1}^{n} A_i$

$A_1 \cup A_2 \cup A_3$ .... $= \bigcup_{i=1}^{\infty} A_i$

$A \cup B = A + B$

Union of finite number of finite sets is a finite set

Union of finite sets with an infinite set is an infinite set

$A \cup A = A$, $A \cup f = A$, $A \cup S = S$

$S \cup f = S$, $f \cup f = f$, If $A \subseteq B$ then $A \cup B = B$

Ex.1 Choose the correct options among the following for any sets A and B.

(a) $P(A) \cup P(B)$ may be equal to $P(A)$

(b) $P(A) \cup P(B)$ may be equal to $P(A \cup B)$

(c) $P(A) \cup P(B)$ must be a subset of $P(A \cup B)$

(d) $P(A) \cup P(B)$ must be equal to $P(A \cup B)$

Sol. (a), (b), (c) are true.

If $A \supseteq B$ then $P(A) \supseteq P(B)$, hence option (a) is true.

If $A = B$ then $A \cup B = A = B$ and $P(A) \cup P(B) = P(A \cup B)$, option (b) is true.

Option (c) is always correct " A and B.

Option (d) is not correct when $A \neq B$. e.q. $A = \{1, 2\}$ and $B = \{1, 4\}$

then $A \cup B = \{1, 2, 4\}$

$P(A) = \{f, \{1\}, \{2\}, [1, 2]\}$

$P(B) = \{f, \{1\}, \{4\}, [1, 4]\}$

$P(A) \cup P(B) = \{f, \{1\}, \{2\}, \{4\}, \{1, 2,\}, \{1, 4\}\}$

$P(A \cup B) = \{f, \{1\}, \{2\}, \{4\}, \{1, 2,\}, \{1, 4\}, \{1, 2, 4\}\}$

So $P(A) \cup P(B) \neq P(A \cup B)$.

INTERSECTION OF TWO SETS

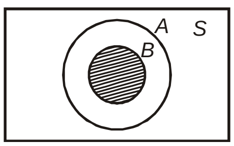
$A \cap B$ (read as 'A intersection B' or 'A cap B' or 'A meet B') is defined as a set containing all the elements common to A and B.

$A \cap B \;=\; \{x : x \in A \text{ and } x \in B\}$

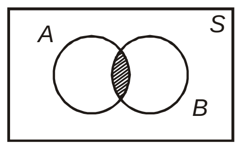
$\;=\; \{x : x \in A \text{ L } x \in B\}$

$x \in A \cap B \Leftrightarrow x \in A \text{ and } x \in B$

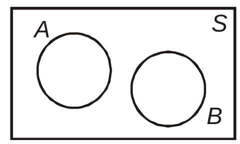
$x \notin A \cap B \Leftrightarrow x \notin A \text{ or } x \notin B$



$A \cap B$



$A \cap B$



$A \cap B = \phi$

$A_1 \cap A_2 \cap A_2 \ldots\ldots\ldots\ldots \cap A_n = \bigcap_{i=1}^{n} A_i = \{x : x \in A_i \;\forall\; i\}$

$A \cap B = AB$

Intersection of finite number of finite sets will be a finite set

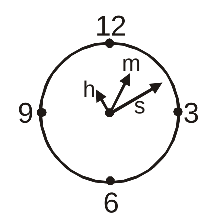
Intersection of finite set with infinite set will be finite set

Intersection of two or more infinite sets may or may not be finite

$A \cap A = A$; $A \cap f = f$; $A \cap S = A$; $S \cap f = f$

$f \cap f = f$; if $A \supseteq B$ then $A \cap B = B$; $(A \cup B) \cap A = A$; $(A \cap B) \cup A = A$

Ex.2 Considering the wall-clock as shown in figure.



Let S = Set of all points in area covered by second's hand in 12 hours.

M = Set of all points in area covered by minute's hand in 12 hours.

H = Set of all points in area covered by hour's hand in 12 hours.

Then pick the correct statement among following.

(a) $S \cup M \cup H = S$ (b) $(S \cap M) \cup H = S$

(c) $S \cap M \cap H = H$ (d) All are correct

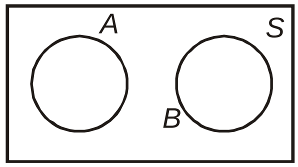
Sol. (a) and (c).

$H \subset M \subset S$ (since hours hand is smallest in length) option (b) is wrong, since $S \cap M = M$ and $(S \cup M) \cup H = M \cup H = M$.

Disjoint Sets

Two sets A and B having no element in common are disjoint or mutually exclusive

$A \cap B = f \Leftrightarrow$ A and B are disjoint.



A and *B* are disjoint

Example:

z^+ and z^- are disjoint

Set of all boys and set of all girls are disjoint

Set of Hindi alphabet and set of English alphabet are disjoint

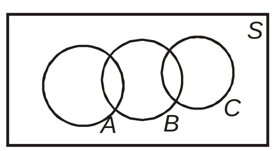
Set of years of birth of adults and set of years of birth of minors are disjoint

Q and Q′ are disjoint

A family of various sets is pairwise disjoint if no two members of this family have a common element.

Note: If A_1, A_2 .... A_n are pairwise disjoint then $A_1 \cap A_2$ .... $\cap A_n = f$,

But if $A_1 \cap A_2$ .... $\cap A_n = f$, A_1, A_2 .... A_n may not be pairwise disjoint.



$A \cap B \cap C = f$; but A and B, B and C are not disjoint.

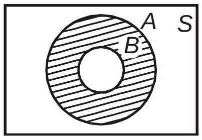
So A, B and C are not pairwise disjoint.

Difference of Two Sets

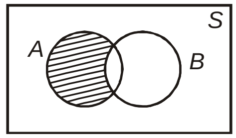
A – B (read 'A minus B') or (relative complement of B in A) is the set of all elements of A which are not elements of B.

$A - B = \{x : x \in A \text{ and } x \notin B\}$

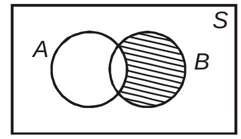
$B - A = \{x : x \in B \text{ and } x \notin A\}$



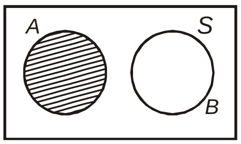
$A - B$ when $B \subset A$



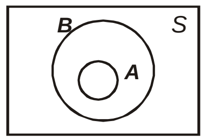
$A - B$



$B - A$



$A - B$, when A and B are disjoint $A - B = A$



$A - B = \phi$ when $A = B$ or $A \subset B$

$x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$

[Difference of two sets is not commutative]

$A - B \neq B - A$

Delete the elements of set B from set A and remaining set is A – B.

$(A - B)$, $(B - A)$ and $(A \cap B)$ are disjoint sets.

$A - B \subseteq A$; $B - A \subseteq B$, $A - f = A$

$A - A = f$

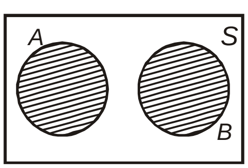
$A - B = A \sim B = A/B = C_A B$ (complement of B in A)

Symmetric Difference of Two Sets

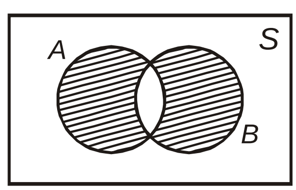
Denoted by A D B or $A \oplus B$, (A direct sum B)

$A \Delta B \quad = (A - B) \cup (B - A)$

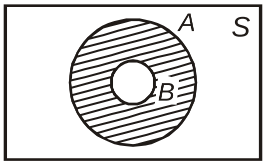
$\qquad\quad = (A \cup B) - (A \cap B)$



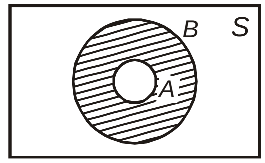
$A \Delta B = A \cup B$ when A and B are disjoint



$A \Delta B = (A \cup B) - (A \cap B)$



$A \Delta B = (A - B)$
when $B \subseteq A$



$A \Delta B = (B - A)$
when $A \subseteq B$

$A \Delta B = B \Delta A$ commutative

$A \Delta B = \{x : x \in A \text{ and } x \notin B\} \cup \{x : x \notin A \text{ and } x \in B\}$