SETS

UNIVERSAL SET

Universal Set

Any set which is the superset of all the sets under consideration is called the universal set (Ω or S or U).

Choice of universal set is not unique, but once chosen it is fixed for that discussion.

Example:

Let A = {a, e, i}; B = {i, o, u}; C = {e, f}

then U = {a, e, i, o, u, f}

or U = {a, e, i, p, o, u, f, g}

or U = Set of all English alphabet.

Ex.1 Mark T/F against the each statement given below:

(a) Every set has at least one proper subset.

(b) If A is a finite, non-void set, having n proper subsets and m subsets then $n - m \in N$.

(c) A = {ϕ, {ϕ}} then cardinal number of P(A) is 4.

(d) $a \subseteq \{a,\{b\},\{c\}\}$

(e) 'Set of all squares in a plane' is a subset of 'all rectangles in the same plane'.

Sol. (a) F (b) F (c) T (d) F (e) T

(a) Void set { }, has no proper subset.

(b) $n - m = -1 \notin N$

(c) $P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi,\{\phi\}\}\}$ so $n\{P(A)\} = 4$.

(d) Since 'a' is not a set hence it cannot be a subset. Every subset is a set in itself.

(e) Each square is also a rectangle hence true.

Ex.2 Match the following columns.

(a) A = letters of word 'ball' (i) $P(A) \subset P(B)$

B = letters of word 'lab'

(b) $A \subset B$ (ii) A and B are incomparable

(c) $A = \{x : \cos x > \frac{-1}{2} \text{ and } 0 \le x \le \pi\}$ (iii) A = B

$B = \{x : \sin x > \frac{1}{2} \text{ and } \frac{\pi}{3} \le x \le \pi\}$

(d) $A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$ (iv) $A \supset B$

$B = \{(x, y) : 0 \le x \le \frac{1}{2} \text{ and } y = 0\}$

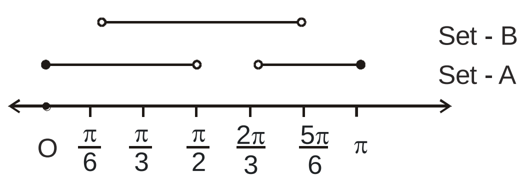
Sol. (a)(iii) (b)(i) (c)(ii) (d)(iv)

(a) A = {a, b, l}; B = {l, a, b} clearly A = B, A and B are comparable P(A) = P(B) so $P(A) \not\subset P(B)$;

$A \not\supset B$. Hence answer (iii) only.

(b) If $A \subset B$ then $P(A) \subset P(B)$; A and B are comparable, $A \ne B$, $A \not\supset B$. Hence answer (i) only.

(c) A and B are as shown on number line.



Clearly $P(A) \not\subset P(B)$

A and B are not comparable.

$A \ne B$ and $A \not\supset B$. Hence answers (ii) only.

(d) A is all points within on a circle of radius 1 and centre (0, 0). B contains only the points lying on x-axis within the circle such that $0 \le x \le$, so clearly $B \subset A$; $B \ne A$, A and B are comparable $P(A) \not\subset P(B)$. Hence only correct answer is (iv).

Intervals as Subsets of R

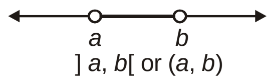
Four type of subsets can be defined on R as given below.

Let a, b $\in$ R, such that a < b

1. Open Interval

(a, b) or] a, b [= {x : a < x < b}

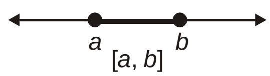
= Set of all real numbers between a and b, not including a and b both.



2. Closed Interval

[a, b] = {x : a £ x £ b}

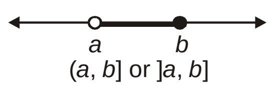
= Set of all real numbers between a and b as well as including a and b both.



3. Open-closed Interval (semi closed or semi open interval)

(a, b] or]a, b] = {x : a < x £ b}

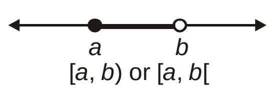
= Set of all real numbers between a and b, a not included but b included.



4. Closed-open interval (semi closed or semi open interval)

[a, b) or [a, b[= {x : a £ x < b}

= Set of all real numbers between a and b including a but excluding b.



Some More Representations on Number Line

Infinite open interval

x > a

$x < b$

Infinite close interval

$x \leq b$

$x^3 a$

$(0, \infty) = R^+$

$(-\infty, 0) = R^-$

$(-\infty, \infty) = R$