# SETS

# SUB SETS

A set is a well-defined group of numbers, objects, alphabets, or any items arranged in curly brackets whereas a subset is a part of the set. The components of sets could be anything such as a group of real numbers, a group of integers, variables, a group of all-natural numbers, constants, whole numbers, etc. Let us discuss subsets in this article with their definition, subset symbol, types, examples and more. Also, learn about sets, subsets and supersets with their difference for more clarity. Subset example; if a set P is a combination of odd numbers and set Q consists of  $\{1,5,7\}$ , then set Q is said to be a subset of set P and is denoted by the symbol Q $\leq$ P. Here P is the superset of Q. Before heading towards the subsets let us take a brief overview of sets and its various types.

#### What is a Subset?

A subset is a subgroup of any set. Consider two sets, A and B then A will be a subset of B if and only if all the components of A are present in B. We can also say that A is contained in B.

To understand the subset definition more clearly, consider a set P such that P comprises the names of all the cities of a country. Another set Q includes the names of cities in your region. Here Q will be a subset of P. This is because all the cities in your region would also be cities of your country; hence, Q is a subset of P. There are only a definite number of distinct/unique subsets for any set, therefore the remaining are irrelevant and repetitive.

#### Example:

A subset as far as our understanding is a set contained in another set. It is like one can pick ice cream from the following flavours: {mango, chocolate, butterscotch}

• You can take any one flavor {mango}, {chocolate}, or {butterscotch},

 Or any two flavors: {mango chocolate}, {chocolate, butterscotch}, or {mango, butterscotch}.

#### What is a Subset Meaning in Maths?

A Set 1 is supposed to be a subset of Set 2 if all the components of Set 1 are also existing in Set 2. In other words, set1 is included inside Set2.

• If Set1= {A, B, C} and Set2= {A, B, C,D,E,F,G,H,I} then we can say that Set1 is a subset of Set2 as all the elements in set 1 are available in set 2.



Set 1 is a subset of Set 2

## Subset Symbol

In the set theory, a subset is expressed by the symbol and addressed as 'is a subset of'. Applying this symbol we can represent subsets as follows:

- $P \subseteq Q$ ; which is read as Set P is a subset of Set Q.
- **Note**: point: A subset can be identical to the set i.e, a subset can contain all the elements that are present in the set.
- **Ex.1** Find whether P is a subset of Q?

P = {set of even digits), Q = {set of whole numbers}

**Sol.** The set of even numbers can be represented as:

 $P = \{0, 2, 4, 6, 8, 10, 12 \dots\}$ 

Similarly, the set of all whole numbers can be represented as:

 $Q = \{0, 1, 2, 3, 4, 5, 6, 7...\}$ 

From the set components of P and Q, we can figure out that the elements of P are present in the set Q. Therefore P is a subset of Q.

**Ex.2** Determine whether P is a subset of Q.

 $P = \{1, 3, 5, 7\}, \qquad Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14....\}$ 

- **Sol.** It can be analyzed from the set elements that P's elements relate to set Q. Hence P is a subset of Q.
- **Ex.3** Conclude whether X is a subset of Y.

X = {All writing material in a stationary workshop}, Y = {Pencils}

**Sol.** Set X involves pens, sketch pens, markers, pencils, notepads, etc. Whereas set Y only carries pencils. So we cannot state that all X's elements are present in Y, which is a requirement for X to be a subset of Y. In this particular case, we can state that Y is a subset of X, but X is not a subset of Y.

 $A = \{Toyota\}, B = \{All brands of cars\}$ 

**Sol.** Set B covers all brands of cars; Maruti Suzuki, Hyundai, Toyota Mahindra, Tata Motors, Mercedes Benz, etc. Moreover, A is a set of Toyota. Then we can say that all elements of A are incorporated into B. Hence, A is a subset of B.

#### All Subsets of a Set

The no. of subsets of a set of any set consisting of all likely sets including its components and the null set. Let us learn with an example.

**Ex.5** Find all the subsets of set  $P = \{2, 4, 6, 8\}$ 

**Sol.** Given,  $P = \{2, 4, 6, 8\}$ 

Number of subsets of P are =

- {}
- {2}, {4}, {6}, {8},
- {2,4}, {2,6}, {2,8}, {4,6}, {4,8}, {6,8},
- {2, 4, 6}, {4, 6, 8}, {2, 6, 8}, (2, 4, 8}
- {2, 4, 6, 8}

#### **Types of Subsets**

Subsets are primarily classified into:

- Proper Subsets
- Improper Subsets

### **Proper Subset**

Any set say "P" is supposed to be a proper subset of Q if there is at least one element in Q, which is not available in set P. That is, a proper subset is one that contains a few components of the original set.

In other words, we can say that if P and Q are unequal sets and all elements of P are present in Q, then P is the proper subset of Q.

It is also termed a strict subset.

#### **Proper Subset Examples**

- **Ex.6** Is P a proper subset of Q where  $P = \{1, 3, 7, 8\}$  and  $Q = \{1, 3, 7, 8\}$ ?
- **Sol.** The answer would be no, P is not a proper subset of Q as both are identical, and Q does not have any unique element, which is not existing in P.
- **Ex.7** Is X a proper subset of Y when  $X = \{1, 6\}$  and  $Y = \{1, 4, 6, 8\}$ ?
- **Sol.** The answer would be yes, X is a proper subset of Y as all the elements of X are present in Y and X is not equal to Y as well.

### **Proper Subset Symbol**

A proper subset is expressed by c and is addressed as 'is a proper subset of'. For example:

P ⊂ Q.

## **Improper Subset**

Suppose two sets, X and Y then X is an improper subset of Y if it includes all the elements of Y. This implies that an improper subset comprises every element of the primary set with the null set.

The improper subset symbol is C. For example:  $P \subseteq Q$ 

- Ex.8 If set P = {2, 3, 5}, then determine the number of subsets, proper subsets and improper subsets.
- **Sol.** Number of subsets: (2), (3), (5), (2,3), (3, 5), (2,5), (2, 3, 5) and  $\varphi$  or {}.

Proper Subsets: 0), (2), (3), (5), (2,3), (3, 5), (2,5)

This can be denoted as  $\{\}, \{2\}, \{3\}, \{5\}, (2,3), (3,5), (2,5) \subset P$ .

Improper Subset: {2, 3, 5}.

This can be denoted as  $\{2, 3, 5\} \subseteq P$ .

## Subset Formula

The set theory symbols were explained by mathematicians to represent the collections of objects. If it is required to select n number of elements from a set including N number of elements, it can be performed in <sup>N</sup>C<sub>n</sub>, ways.

#### **Proper Subset Formula**

If a set holds "n" elements, then the number of the subset for the given set is 2<sup>n</sup> and the number of proper subsets of the provided subset is calculated by the formula 2<sup>n</sup> - 1.

**Ex.9** For a set P with the elements,  $P = \{1, 2\}$ , determine the proper subset?

Sol. The proper subset formula is  $2^n - 1$  (where n is the number of elements in the set)  $P = \{1, 2\}$ 

Total number of elements (n) in the set=2

Hence the number of proper subset= $2^2 - 1 = 3$ 

Therefore the total number of proper subsets for the given set is {}, {1}, and {2}.

**Ex.10** For the given set determine the power set?

**Sol.** Set  $Y = \{2,3,6\}$ 

Total number of components in the set Y=3

The power set of Y is:

 $P(Y) = \{\}, \{2\}, \{3\}, \{6\}, \{2,3\}, \{3,6\}, \{2,6\}, \{2,3,6\}$ 

 $P(Y)=2^n$ 

Substituting n=3

P(X) = 23 = 8

#### How to Represent Subsets?

We are quite clear with what a subset is, now let us check some of the representations for the same. A subset, like any other set, is addressed with its elements inside curly braces.

Hence, consider two sets, X and Y:

•  $X \subseteq Y$ 

The above notation indicates that X is a subset of Y.

•  $X \subsetneq Y$ 

Here the notation indicates that X is not a subset of Y.

•  $X \subset Y$ 

If X is a proper subset of Y, then it is expressed by the above notation.

•  $X \subsetneq \mathcal{Y}$ 

If X is not a proper subset of Y, then we address it by the above notation.

# **Properties of Subsets**

Some of the important properties of subsets are as follows:

• Every set is said to be a subset of the provided set itself. Either the set is finite or infinite, a set itself will be taken as the subset of itself.

For example, for a finite set  $A = \{3,6\}$ , all the possible subsets for the given data is:

 $A = \{\}, \{3\}, \{6\}, \{3,6\}.$ 

As you can recognize that, we have included a subset with identical elements as the initial set to satisfy the property.

• We can state, an empty set is regarded as a subset of every set.

For example, take a finite set  $B = \{(a, b), so all the possible subsets of this set are:$ 

For example, take a finite set  $B = \{a, b\}$ , so all the possible subsets of this set are:

 $A = \{\}, \{a\}, \{b\}, \{a, b\}$ 

- If P is a subset of Q, then we can state that all the elements in P are available in Q.
  Consider a set P= {2, 6, 9} and another set as Q = {2, 3, 4, 5, 6, 7, 8, 9}. Here we can say P is a subset of Q as all the elements of P are present in Q.
- If set A is a subset of set B then we can assume that B is a superset of A.
- There are 2<sup>n</sup> subsets and 2<sup>n</sup> 1 proper subsets for a provided set of data.

#### **Representation of Subsets through Venn Diagrams**

To understand the links between different sets, the Venn diagram is the most proper tool to reflect logical connections among certain sets. They are employed abundantly for the

design of sets, more importantly for finite sets. A Venn diagram indicates the sets as the area inside a circular target with the elements as points inside the area.

As subsets usually involve two sets, we can easily practice a Venn diagram to illustrate and visualize them.

**Ex.11** For the set  $X = \{1, 3, 6\}$  and set  $Y = \{1, 3, 6, 9, 12, 15, 18\}$  draw the venn diagram.

**Sol.** The Venn diagram illustration of sets X and Y are as follows:



Venn Diagram Representation Of Subsets

As we can recognize from the diagram that X, surrounded by a region denoted by its set, is a portion of region Y. Each area has its elements expressed as points inside the region.

#### **Difference between Proper Subset and Superset**

A common element of confusion among the students in set theory at the beginning is the difference between a proper subset and a superset. So let us understand them with an example:

As discussed in the article, a set P is a proper subset of Q if Q possesses at least one component that is not present in P. It is indicated by the symbol  $\subset$ .

Whereas Q will be the superset of P if and only if all the elements existing in P are a part of Q, which states that Q is greater in size when compared to P. If P denotes the proper subset of Q, then Q will be the superset of P. Denoted by the symbol  $\supseteq$ .

**Ex.12** For the given two sets  $X = \{2, 4, 6, 8\}$  and  $Y = \{2, 4, 6\}$  check if X is a superset of Y.

Sol. For X to be a superset of Y, it requires holding all the elements present in set Y. As we can notice from the given data that X has all the elements that are present in Y. Also, Y is a proper subset of X; hence, X is Y's superset.

We hope that the above article on Subsets is helpful for your understanding and exam preparations. Stay tuned to the Testbook app for more updates on related topics from