SETS

SETS & THEIR REPRESENTATION

The concept of **set** is the basis of the modern Mathematics. It is widely used in various branches of Mathematics.

`Set' was used for the first time by a German Mathematician 'George-cantor'. He defined set as "Any collection into a whole of definite and distinct objects of our intuition or thought". This definition of set was discussed and modified to the most acceptable form as "A set is any collection of distinct and distinguishable objects of our intuition or thought".

In this chapter, the emphasis is on developing the graphical approach among students while solving a problem, from the very beginning. The use of Venn diagrams makes many problems very simple and it should be put to use as frequently as possible.

The concept of **relation** is very useful to understand a function. If function or not as a function can only be understood the concept of relation is clear.

'aRb' means 'a' is R-related to b', where R may be any given relation between a & b.

The concept of **function** lays the foundation of the study of the most important branch 'calculus' of mathematics. The word 'function' is derived from a Latin word meaning 'operation'. Function is also called mapping.

SET : "Set is a well-defined collection of distinct objects"

The objects of a set have a common property. An object having this property belongs to this set and another object not possessing this property does not belong to that set.

For example, the collection of books written by Shakespere is a set, but the collection of interesting books written by Shakespere is not a set, since a book found interesting by one person may not be liked by another.

Example :

The set of all known planets in solar system, set of days in a week, set of all whole numbers, set of consonants in English alphabet etc.

Ex.1 Choose the collection of objects, among the following, that are sets.

1. The collection of all students of Aakash Institute.
2. The collection of most talented Artists of India.
3. The collection of bright students at IIT Kanpur.
4. The collection of all prime-ministers of India.
5. The collection of all lucky numbers.
6. The collection of Indian states.
7. The collection of all tasty dishes.
8. The collection of all secular nations.

Sol. 1, 4, 6, 8 are sets as there is no ambiguity about their members. 2 and 3, 6 and 7 do not represent sets as there is no definite yardstick for being most talented, bright, lucky or tasty. Different people shall address to these terms differently.

Set-Notations

A set is usually denoted by capital letters A, B, C,... etc. whereas its members or elements or objects are denoted by lowercase letters such as a, b, c,....etc.

The greek symbol is used to denote the phrase 'belongs to'. Symbol is called membership relation.

x A $\Rightarrow$ 'x belongs to A' or 'x is an element of A' or 'x is a member of A' or 'x is an object of A'.

x A $\Rightarrow$ 'x does not belong to A'

e.g. a set of English alphabet.

6 set of English alphabet.

Representation of a Set

A set can be represented by two methods

1. Roster form or tabular form
2. Set builder form or rule method.

Roster or Tabular Form

Here the elements of set are listed seperated by commas within braces or curly brackets {}. Here order of listing is immaterial and no element is repeated.

For example, the set A of all single digit natural numbers is written as

A = {1, 2, 3, 4, 5, 6, 7, 8, 9} or A = {1, 3, 5, 2, 6, 4, 9, 8, 7} (order is immaterial)

Set-Builder Form:

Here we choose a variable (say x), which represents each element of the set satisfying a particular property. Inside the bracket, x is followed by symbol: (or ; or vertical line '|' or oblique line '/' followed by the property or properties, possessed by each element of set. For example, the set A of all even integers less than 10 is written as

A = {x : x is an even integer less than 10}

= {x | x is an even integer less than 10}

= {x ; x is an even integer less than 10}

= {x / x is an even integer less than 10}

The symbol following x is read as 'such that'.

The roster form of A is written as

A = {0, 2, 4, 6, 8}

Note : '0' is an even integer

Set builder form is also called, rule method, property method or symbolic method.

Ex.2 Write each of the following into another form of set writing.

(a) A = {x / x N and x 6}

(b) $B = \left\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}\right\}$

(c) C = {1, – 1, i, – i}

(d) D = {2, 4, 8, 16, 32}

(e) $E = \{x : x^2 - 5x + 6 = 0\}$

(f) F = {x | x is a letter of word IITJEE}

(g) $G = \{n^3 - n^2 : n \ \ N \text{ and } 2n \ \ 4\}$

(h) H = {1, 8, 27, 64,..........,10)

Sol. (a) A = {1, 2, 3, 4, 5, 6}

(b) Each element is of the form $\frac{2n-1}{2n}$ hence

$B = \{x : x = \frac{2n-1}{2n}, n \ \ N, n \ \ 4\}$

(c) 'C' is a set of fourth roots of unity

Hence $C = \{x : x^4 = 1\}$

(d) 2, 4, 8, 16, 32 are clearly of the form 2^n, where n is a natural number less than 6. so

$D = \{x : x = 2^n; n\ N, n < 6\}$

(e) The roots of given equation must form the solution, hence E = {2, 3}

(f) No element has to be repeated, hence F = { I, T, J, E}

(g) G = {4, 18, 48}

(h) All the listed numbers are cube of natural numbers. So

$H = \{x : x = n^3, n \ \ N, n\ 10\}$

Standard Notations for Sets of Numbers

	Set of all	Symbol	i.e.
1.	Natural number	N	N = {1, 2, 3.......}
2.	Integers	Z or I	Z or I = {.......–3, –2, –1, 0, 1, 2, 3,....}
3.	(a) Positive integers	Z^+	Z^+ = {1, 2, 3,......}
	(b) Negative integers	Z^-	Z^- = {.....–3, –2, –1}
4.	Integers excluding 0	I_0	I_0 = {±1, ±2, ±3........}

5.	Even integers	E	E = {0, ±2, ± 4........}
6.	Odd integers	O	O = {±1,±3,±5........}
7.	Rational numbers	Q	Q = {x : x =$\frac{p}{q}$, p and q are integers, q≠0}
8.	Non-zero rational numbers	Q_0	Q_0= {x : x ∈Q, x≠0}
9.	Positive rational numbers	Q^+	Q^+ = {x : x ∈Q, x > 0}
10.	Real numbers	R	Here all rational and irrational numbers are included
11.	Non-zero real numbers	R_0	R_0 = {x : x ∈ R, x≠ 0}
12.	Positive real number	R^+	R^+ = {x : x ∈ R, x > 0}
13.	Complex numbers	C	C = {a + ib; a, b∈ R and i =$\sqrt{-1}$ }
14.	Non-zero complex number	C_0	C_0 = {x : x ∈ C, x≠ 0}
15.	Natural numbers less than or equal to K, where K is positive integer	N_k	N_k = {1, 2, 3, 4, ...k}
16.	Whole numbers	W	W = {0, 1, 2, 3,......}

R is a subset of C $(R \subset C)$. Irrational numbers cannot be written in $\frac{p}{q}$ form.

Non-repeating and non-terminating decimals are called irrational.

e.g. $\sqrt{2}, \sqrt[5]{3}, \pi, e, \log_2 10$