SETS

POWER SETS

Power Set

The set of all subsets of a set A is called power set of A and is denoted by P(A) or 2^A.

$P(A) = \{x : x \subseteq A\}$

$x \in P(A) \Leftrightarrow x \subseteq A$

$\phi \in P(A)$ and $A \in P(A)$

Example :

$A = \{1, 2, 3\}$

$P(A) = 2^A = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$n(A) = 3$ so $n(2^A) = 2^3 = 8$

(i) If A has n elements then its power set P(A) contains 2^n elements. $nP(A) = 2^n$.

(ii) ϕ and A both belong to P(A).

(iii) If $A = \phi$ then $P(A) = \{\phi\}$ is a singleton set.

(iv) If $A = \{t\}$ then $P(A)$ $\{\phi, \{t\}\}$ is a pair set.

(v) If cardinal number of set A is n then total number of subsets of $P(A) = 2^{2^n}$ and proper subsets $2^{2^n} = -1$.

(vi) If $A \subseteq B \Rightarrow P(A) \subseteq P(B)$

Ex.1 $A = \{(x, y) : y^2 = x, x \in R\}$ and $B = \{(x, y) : y =, x \in R\}$. Choose the correct option/options among the following.

(a) $A = B$

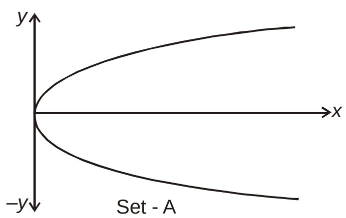
(b) $B \subset A$ and $A \subseteq B$

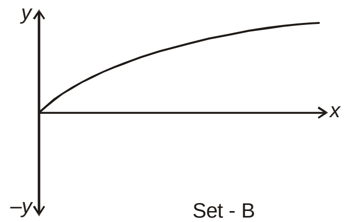
(c) A and B are comparable sets.

(d) A and B both are infinite sets.

Sol. (c) and (d)

Set A and Set B include all the points lying on the respective curves below. Clearly $A \neq B$, $B \subset A$, so A and B are comparable; $A \not\subseteq B$ and both are infinite sets as infinite number of points satisfy each.





Ex.2 Prove that if A has n elements then its power set has 2^n elements.

Sol Methods of selecting r things from n different things is given by

$${}^{n}C_{r} = \frac{n(n-1)(n-2)\ldots \text{ uptor terms}}{\underline{r}}, \text{ where}$$

$\lfloor r = 1.2.3 \ldots r$.

The number of subsets of A having no element = ${}^{n}C_{0} = 1$ i.e. ϕ

The number of subsets of A having one element = ${}^{n}C_{1}$

The number of subsets of A having two elements = ${}^{n}C_{2}$

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The number of subsets of A having n elements = ${}^{n}C_{n}$

Thus total number of subsets of A $= {}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots\ldots + {}^{n}C_{n}$

$= (1+1)^n = 2^n$ (by binomial theorem)