SETS

FINITE & INFINITE SET

Finite Set is a set with a finite number of elements while an infinite set is a set with an infinite number of elements. Finite and infinite sets are counted under the various types of sets. Finite elements/components point towards countable data on the other hand infinite elements/components means uncountable data or data that cannot be counted. A similar concept applies to the finite set and infinite set. In this article, amongst the various types of sets (empty set, equivalent set, subsets, singleton set, power set, universal set, disjoint set, superset, equal set, finite and infinite set), we will talk about the finite and infinite sets with examples.

What are Finite and Infinite Sets?

Finite Set: A set with a finite number of elements is named a finite set. We can also understand these sets have a definite/countable number of elements.

Example of a finite set:

Set P = {4,8,12,16, 20) is a finite set, as it has a finite number of elements.

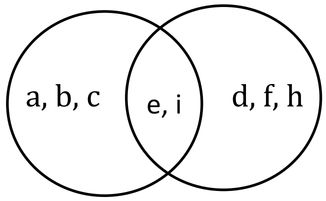
Infinite Set: This is exactly opposite of the finite set. This implies that the infinite set will have an infinite number of elements/components.

Example of an infinite set:

Set of all prime numbers, a set of all even numbers, or a set of all odd numbers are examples of an infinite set.

Venn Diagram of Finite and Infinite Sets

Finite sets and infinite sets in mathematics are exactly opposite to one another or can say are completely dissimilar from each other as per their definition. Now let us understand these sets with help of the Venn diagram. A Venn diagram includes overlapping closed circles. Each circle inside a diagram represents a set that shows the relationship between different sets.



Consider the above Venn diagram. It consists of:

Set P = (a, b, c, e, i}

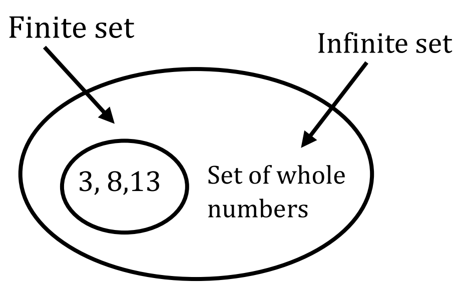
Set Q = {e, i, d, f, h}

$P \cup Q$ = {a, b, c, e, i, d, f, h}

$P \cap Q$ = {e, i}

Like the sets, P and Q are finite i.e n(P) = 5 and n(Q) = 5, therefore the result of union of sets ($P \cup Q$) and intersection of sets ($P \cap Q$) are also finite.

Now consider the below Venn diagram:



The diagram shows that there are two sets, wherein the inner set with data {3, 8, 13} represents a finite set, however, the outer set that represents a set of whole numbers is an infinite set. This is infinite as there are an infinite number of elements in the set.

Properties of Finite and Infinite Sets

Now that we know what is a finite set and infinite set using the Venn diagram as well. Let us move toward the related properties.

If a given set is finite, then the below properties exist:

- The subsets of a finite set are always finite.
- The union as well as the intersection of the given finite set results in a finite set.
- For a finite set, the power set is also definite.
- For finite sets, the cartesian product is also finite.
- The cardinal number for a finite set is a definite number.

If a given set is infinite, then the below properties exist:

- The union result of infinite sets is always infinite.
- For the given infinite sets the power set is also infinite.
- The resultant superset of an infinite set is also infinite.
- The subset for an infinite set can or cannot be infinite. This depends on the number of elements in a given subset.

Difference between Finite and Infinite Sets

Finite and infinite sets are the basic classification of sets, i.e. either a set of elements is finite or infinite. Moving forward, let us understand the difference or comparison between them:

FINITE SETS	INFINITE SETS
Elements: The number of elements is countable for such sets.	**Elements:** The number of elements is uncountable for these types of sets.
Subset: A subset for these sets contains countable elements i.e. the subset is finite.	**Subset:** For these sets, a subset may or may not be countable.
Union of sets: The components in the union of such sets are countable/ finite.	**Union of sets:** The union result for such sets is also infinite.
Power set: The power set holds a finite number of elements.	**Power set:** The power set has an infinite number of elements.

Cardinality: The cardinality for these sets is equal to the number of elements in the set. That is, the cardinality of a set with n elements is equal to n.	**Cardinality:** The cardinality for these sets is equal to infinity (∞). This is due to the fact that the number of elements is uncountable.
Example: Number of days in a week / months, months in a year, seasons in the year, set of prime numbers less than 50,etc.	**Example:** A set of all whole numbers, set of all-natural numbers, set of all real numbers, etc.

Ex.1 Check if an empty set is a finite set or not.

Sol. An empty set as known to us has no elements in it and is represented by the notation {} or Ø.

The empty sets have zero elements and zero is definite, therefore an empty set is a finite set.

Ex.2 For the given sets where X = {3, 6, 9, 12) and Y = {6, 12, 18, 24), comment on the nature of union of these sets.

Sol. Given: X = {3, 6, 9, 12) and Y = {6, 12, 18, 24}

The union of X and Y is:

$X \cup Y=\{3, 6, 9, 12, 18, 24\}$

As we can see that the sets X and Y have a definite number of elements and so is the union of X and Y.

Ex.3 A set with data as {...,-5,-4,-3, -2, -1, 0, 1, 2, 3, 4, 5...} is finite or infinite, justify your answer?

Sol. Let us name the set as Q, therefore Q= {...,-5,-4,-3, -2, -1, 0, 1, 2, 3, 4, 5...}.

You can verify that the set has no beginning or ending point, hence the number of elements in the set Q is uncountable. Therefor set Q is an infinite set.

Ex.4 Comment on the nature of the given below sets:

A= {p: p N, 2<p<13}

B= {q: q ∈ W, q<21}

C= {x: X∈ R and x+6>15}

Sol. Given that:

A= {p: p N, 2<p<13}

B= {q: q ∈ W, q<21}

C= {x: x ∈ R and x+6>15}

Let us check these sets one by one:

A= {p: p N, 2<p<13}

Set A includes natural numbers between 2 to 13, which are countable hence set A is a finite set.

B= {q: q∈W, q<21}

Set B holds whole numbers which are less than 21. The elements are countable making it finite.

C= {x: x ∈ R and x+6>15}

The set contains real numbers that are greater than 9{x+6>15, x+6-6>15-6, x>9 }. In this set, there is no endpoint for the set. Hence C is an infinite set.

Ex.5 If, n(A) = 10. Then find the cardinality of power set of A?

Sol. Concept:

1. Cardinality of a set A is defined as the no. of elements present in the set A, denoted by n(A).

2. Power set: Let A be a non-empty set. Then, P(A) = {B | B C A} is called as the power set of A.

3. For any non-empty set A with n(A) = x. Then the total no. of subsets of set A is given by: 2x.

i.e n(P(A)) = 2x.

Calculation:

Given: n(A) = 10, here x = 10.

n(P(A)) = 210

Hence, the cardinality of power set of A is 210