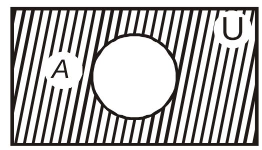
SETS

COMPLEMENT OF SETS

Complement of a Set

If 'A' be a set and U be the universal set such that $A \subset U$ then complement of set A is denoted by A' or A^C or C (A) or U – A

$A' = A^C = C(A) = U - A = \{x : x \in U \text{ and } x \notin A\}$



$U - A$, or A^c or A' or $C(A)$

if $\quad x \in A \Leftrightarrow x \notin A'$

$x \in A' \Leftrightarrow x \notin A$

$U' = \phi$; $\phi' = U$; $A \cup A' = U$, $A \cap A' = \phi$

Ex.1 If $A = \{5, 6, 7, 8\}$; $B = \{3, 9, 8, 10\}$ and $S = \{1, 2, 3, 4....... 10\}$ then,

(a) $(A \cup B)' = \{1, 2, 3, 4\}$ (b) $(A \cap B)' = \{8\}$

(c) $(A' \cap B)' = (A \cap B)'$ (d) None of these is true

Sol. (d)

$A \cup B = \{3, 5, 6, 7, 8, 9, 10\}$ hence $(A \cup B)' = \{1, 2, 4\}$, (a) is wrong.

$A \cap B = \{8\}$ hence $(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$, (b) is wrong.

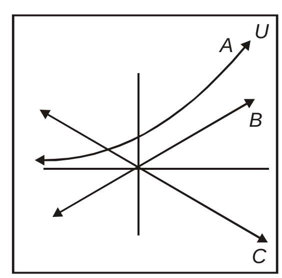
$(A' \cap B)' = \{1, 2, 4\} \neq (A \cap B)'$, (c) is wrong.

Hence answer (d) is correct.

Ex.2 If $A = \{(x, y) : y = e^x; x \in R\}$ $U = \{(x, y) : x, y \in R\}$

$B = \{(x, y) : y = x; x \in R\}$

$C = \{(x, y) : y = -x; x \in R\}$



Choose the correct statement/s among the following :

(a) $(A \cap B)' = \phi$ (b) $(A \cap B \cap C)' = \phi$

(c) $A - B = \phi$ (d) $A \Delta B = A \cup B$

Sol. (d)

Set A, B and C are the points on the curves as shown in adjacent diagram. Clearly

$A \cap B = \phi$

so $(A \cap B)' = U$, option (a) is wrong.

Similarly $A \cap B \cap C = f$ as there are no common points to all the three curves.

$\therefore\ (A \cap B \cap C)' = U$.

Option (b) is wrong.

From figure it is clear that $A - B = A$, since A and B are disjoint sets. Option (c) is wrong.

$$A \Delta B = (A - B) \cup (B - A)$$
$$= A \cup B$$

ALGEBRA OF SETS

1. Idempotent Laws : For any set A, we have

(a) $A \cup A = A$

(b) $A \cap A = A$

2. Identity laws : For any set A, we have

(a) $A \cup \phi = A$

(b) $A \cap \phi = \phi$

(c) $A \cup U = U$

(d) $A \cap U = A$

3. Commutative laws : For any two sets A and B, we have

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

4. Associative laws : For any three sets A, B and C, we have

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

5. Distributive laws : For any three sets A, B and C, we have

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. Demorgan's laws **:** For any three sets A, B and C, we have

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

(c) $A - (B \cup C) = (A - B) \cap (A - C)$

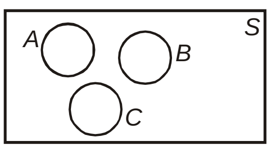
(d) $A - (B \cap C) = (A - B) \cup (A - C)$

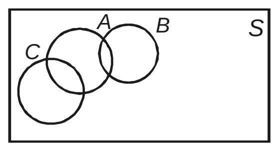
$(A')' = A$ (for any set A)

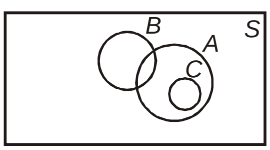
$P(A) \cap P(B) = P(A \cap B)$

$P(A) \cup P(B) \subseteq P(A \cup B)$

Ex.3 Shade $(A \cup B) \cap (A \cup C)$ in the following diagrams

(i)


(ii)


(iii)


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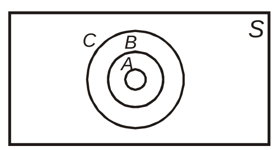
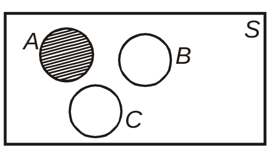
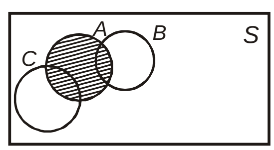
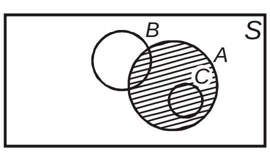
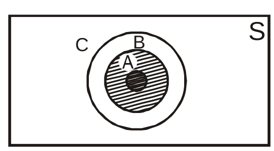


Sol.

(i)


(ii)


(iii)


(iv)


SOME THEOREMS

For any sets A, B and C, we have

1. $A - B = A \cap B'$
2. $A \cup B = B \Leftrightarrow A \subseteq B$
3. $A \cap B = A \Leftrightarrow A \subseteq B$
4. $(A - B) \cup B = A \cup B$
5. $A - (B \cup C) = (A - B) \cap (A - C)$

   $A - (B \cap C) = (A - B) \cup (A - C)$

Some more operations on sets.

$A \subseteq A \cup B$; $A \cap B \subseteq A$; $(A - B) \cap B = \phi$

$A \subseteq B \Leftrightarrow B' \subseteq A'$; $A - B = B' - A'$

$(A \cup B) \cap (A \cup B') = A$; $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

$A - (A - B) = A \cap B$; $A - B = B - A \Leftrightarrow A = B$

$A \cup B = A \cap B \Leftrightarrow A = B$; $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Some Basic Results about Cardinal Numbers

If A, B and C are finite sets and U is finite universal set, then we have

1. $n(A') = n(U) - n(A)$
2. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. If $n(A \cup B) = n(A) + n(B) \Leftrightarrow A \cap B = \phi$ i.e. A and B are disjoint.
4. $n(A \cap B') = n(A) - n(A \cap B)$
5. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
6. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
7. $n(A - B) = n(A) - n(A \cap B)$
8. $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$
9. $n(A \Delta B) = n(A \cup B) - n(A \cap B)$

10. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$

11. If $A_1, A_2 \ldots A_n$ are pairwise disjoint sets then, $n(A_1 \cup A_2 \cup \ldots \cup A_n)$
$= n(A_1) + n(A_2) + \ldots + n(A_n)$

12. Number of elements belonging to exactly two of sets A, B and
$C = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3.\, n(A \cap B \cap C)$

13. Number of elements belonging to atleast two of sets A, B and
$C = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2.\, n(A \cap B \cap C)$

14. Number of elements belonging to atmost two of sets A, B and $C = n(A \cup B \cup C) -$
$n(A \cap B \cap C)$

15. Numbers of elements belonging to exactly one of the sets A, B and
$C = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(A \cap C) - 2n(B \cap C) + 3\, n(A \cap B \cap C)$