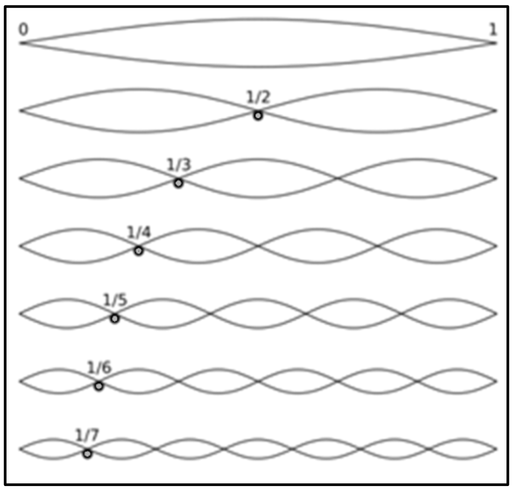
STRUCTURE OF ATOM

TOWARDS QUANTUM MECHANICAL MODEL OF THE ATOM

- **DeBroglie's hypothesis (dual nature of matter):**

  deBroglie proposed that like light, electron possess dual nature i.e., behave like both as material particle and as wave. The concept of dual character of metal evolved a wave mechanical theory of atom according to which electrons. protons and even atoms possess wave properties when moving.



Derivation of deBroglie Relationship:

For a photon,

$$E = \frac{hc}{\lambda} \qquad ......(i)$$

$$E = mc^2 \qquad ......(ii)$$

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} = \frac{h}{p}$$

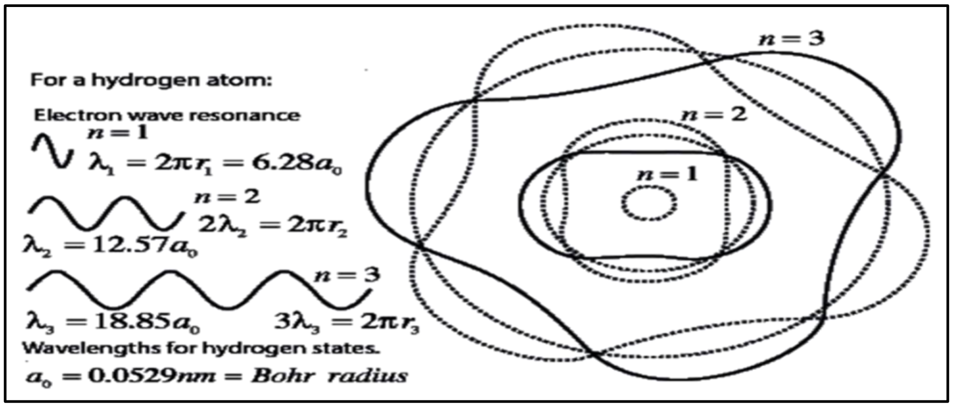
He concluded that as electromagnetic radiation have some associated mass or associated momentum, in the same way every moving particle of mass 'm' and velocity 'v' is associated with waves and these associated waves are known as matter waves or deBroglie's waves.

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Experimentation verification of deBroglie's hypothesis: Given by Davission and Germer.

They observed that a beam of electrons are diffracted by nickel crystal just like the x-rays. Moreover, it wavelength of the electrons determined by the diffraction experiments are equal to the values calculated from deBroglie's relations ship.

DERIVATION OF BOHR'S POSTULATE OF QUANTISATION OF ANGULAR MOMENTUM FROM DEBROGLIE'S EQUATIONS-



According to deBroglie moving electron is associated with waves must be completely in phase and therefore only those orbits are possible where circumference of the orbit is integral multiple of λ i.e.,

$$2\pi r = n\lambda$$

, where n is the no. of waves

$$2\pi r = \frac{\mathrm{nh}}{\mathrm{mv}} \quad \Rightarrow \quad \mathrm{mvr} = \frac{\mathrm{nh}}{2\pi}$$

From the above expression it can be commented that

no of waves in a shell wave = shell no.

Calculation of deBroglie wavelength if K.E. of the particle is E:

$$\mathrm{E} = \frac{1}{2}\mathrm{mv}^2$$

$$2\mathrm{E} = \mathrm{mv}^2$$

$$2\mathrm{mE} = \mathrm{m}^2\,\mathrm{v}^2 = \mathrm{p}^2$$

$$\mathrm{p} = \sqrt{2\mathrm{mE}}$$

$$\frac{\mathrm{h}}{\lambda_{\mathrm{dB}}} = \sqrt{2\mathrm{mE}}$$

$$\lambda_{\mathrm{dB}} = \frac{\mathrm{h}}{\sqrt{2\mathrm{mE}}}$$

If a charge particle at rest (having charge 'q') is accelerated by potential difference 'V' volt then

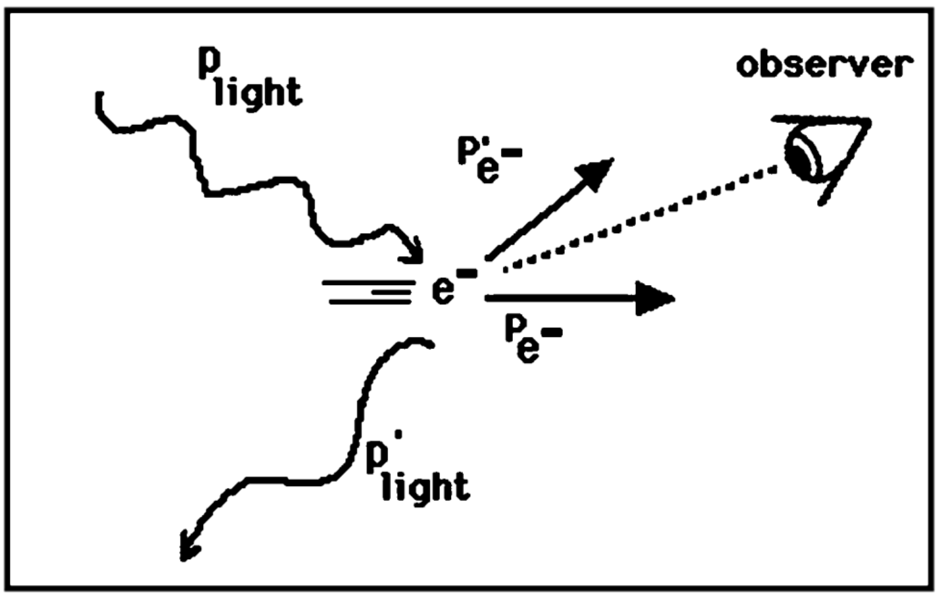
$$\lambda_{dB} = \frac{h}{\sqrt{2mqv}},$$

$$\lambda_{dB} = \sqrt{\frac{150}{V}}\,Å \quad \text{(Only for electron)}$$

Ex. Calculate λ_{dB} of electron have K.E. 3eV

Sol. $\lambda_{dB} = \sqrt{\frac{150}{3}} = \sqrt{50}\ Å$

The Uncertainty Principle



The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of the uncertainties of these two measurements. There is likewise a minimum for the product of the uncertainties of the energy and time.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

The uncertainty principle contains implications about the energy that would be required to contain a particle within a given volume. The energy required to contain particles comes from the fundamental forces, and in particular the electromagnetic force provides the attraction necessary to contain electrons within the atom, and the strong nuclear force provides the attraction necessary to contain particles within the nucleus. But Planck's constant, appearing in the uncertainty principle, determines the size of the confinement that can be produced by these forces. Another way of saying it is that the strengths of the nuclear and electromagnetic forces along with the constraint embodied in the value of Planck's constant determine the scales of the atom and the nucleus.

HEISENBERG UNCERTAINTY PRINCIPLE

Bohr's theory considers an electron as a material particle. Its position and momentum can be determined with accuracy. But, when an electron is considered in the form of wave as suggested by de-Broglie, it is not possible to ascertain simultaneously the exact position and velocity of the electron more precisely at a given instant since the wave is extending throughout a region of space.

In 1927, Werner Heisenberg presented a principle known as Heisenberg uncertainty principle which states as: "It is impossible to measure simultaneously the exact position and exact momentum of a body as small as an electron."

The uncertainty of measurement of position, Δx, and the uncertainty of momentum Δp or $m\Delta v$, are related by Heisenberg's relationship as: $(p = mv, \Delta p = m\Delta v)$

$$\Delta x \,.\, \Delta p \geq \frac{h}{4\pi} \quad \text{or} \quad \Delta x.\, m\Delta v \geq \frac{h}{4\pi} \quad \text{or} \quad \Delta x.\, \Delta v \geq \frac{h}{4\pi m}$$

where h is Planck's constant.

$$\Delta x\, \Delta v = \text{uncertainty product}$$

For an electron of mass m $(9.10 \times 10^{-28}$ g), the product of uncertainty is quite large.

$$\Delta x \,.\, \Delta v \geq \frac{6.624\times10^{-27}}{4\pi m}$$

$$\geq \frac{6.624\times10^{-27}}{4\times3.14\times9.10\times10^{-28}}$$

$$= 0.57 \text{ erg sec per gram approximately}$$

When $\Delta x = 0$, $\Delta v = \infty$ and vice-versa.

In the case of bigger particles (having considerable mass), the value of uncertainty product is negligible. If the position is known quite accurately, i.e., Δx is very small, Δv becomes large and vice-versa.

- In terms of uncertainty in energy ΔE, and uncertainty in time Δt, this principle is written as,

- Heisenberg replaced the concept of definite orbits by the concept of probability. According to Heisenberg $\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$ we can only define the probability of finding electrons around the nucleus.

Ex. Why electron cannot exist inside the nucleus according to Heisenberg's uncertainty principle?

Sol. Diameter of the atomic nucleus is of the order of 10^{-15} m

The maximum uncertainty in the position of electron is 10^{-15} m.

Mass of electron = 9.1×10^{-31} kg.

$$\Delta x . \Delta p = \frac{h}{4\pi}$$

$$\Delta x \times (m.\Delta v) = \frac{h}{4}\pi$$

$$\Delta v = \frac{h}{4\pi} \times \frac{1}{\Delta x \cdot m} = \frac{6.63 \times 10^{-34}}{4 \times \frac{22}{7}} \times \frac{1}{10^{-15} \times 9.1 \times 10^{-31}}$$

$$\Delta v = 5.80 \times 10^{10}\ ms^{-1}$$

This value is much higher than the velocity of light and hence not possible.

DE BROGLIE RELATIONSHIP & HEISENBERG'S UNCERTAINTY PRINCIPLE

Ex. The mass of a particle is 1 mg and its velocity is 4.5×10^5 cm per second. What should be the wavelength of this particle if h = 6.652×10^{-27} erg second.

(1) 1.4722×10^{-24} cm **(2)** 1.4722×10^{-29} cm

(3) 1.4722×10^{-32} cm **(4)** 1.4722×10^{-34} cm

Sol. Given that

$$m = 1\ mg = 1 \times 10^{-3}\ g$$

$$c = 4.5 \times 10^5\ cm/sec.$$

$$h = 6.652 \times 10^{-27}\ erg\ sec.$$

$$\therefore \quad \lambda = \frac{h}{mc} = \frac{6.652 \times 10^{-27}}{1 \times 10^{-3} \times 4.5 \times 10^5} = \frac{6.652 \times 10^{-29}}{4.5} = 1.4722\ cm \times 10^{-29} cm$$