STRUCTURE OF ATOM

DEVELOPMENTS LEADING TO THE BOHR'S MODEL OF ATOM

❖ BOHR'S MODEL

◆ POSTULATES OF BOHR'S MODEL

1. An atom consists of centrally located small, dense, positively charged nucleus and electrons are revolving around the nucleus in circular paths known as circular orbits and coulombic force of attraction between nucleus and electron is balanced by centrifugal force of the revolving electron.
2. Out of the infinite circular orbits only those circular orbits are possible in which angular momentum of electron is integral multiple of $h/2\pi$ i.e., angular momentum of an electron can have fixed values like $\frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3\ h}{2\pi}$ etc. i.e., angular momentum of electron is quantized.

$$mvr = \frac{nh}{2\pi}$$

m and v are mass and velocities of electron respectively and r is radius of orbit and n is integer which is later related with orbit number and shell number and h is Plank's constant.

3. The energy of these circular orbits have fixed values and hence electron in an atom can have only certain values of energy. It is characteristic of an orbit and it cannot have any orbit value of its own. And the refore energy of an electron is also quantized.
4. As long as electrons remains in these fixed orbits, it doesn't lose energy i.e., energy of an electron is stationary (not changing with time) and therefore these orbits are known as allowed energy levels or stationary states and this explains the stability of atom.
5. The energy levels are designated as K, L, M, N and numbered as 1, 2, 3, 4 etc from nucleus outwards and as the distance of the shell's or energy level from the nucleus increases the energy of the energy level also increases i.e.,

$$E_N > E_M > E_L > E_k$$

6. The emission or absorption of energy in the form of radiations can only occur when an electron jumps from one stationary states to other.

$$\Delta E = E_{higher} - E_{lower}$$

$$\Delta E = h\nu$$

where hν is the energy of absorbed photon or emitted photon which corresponds to the difference in energy levels. Energy is absorbed when electron jumps from lower energy level (normal state)

to higher energy level (exited, unstable state) and energy is emitted when electrons jump from higher energy level to lower energy level.

- Bohr's model is applicable for a one electron species only, like H, He^{+}, Li^{+2}, Be^{+3} etc.
- Derivation of Radius of different orbits in one electron species (using Bohr's model) :

$$mvr = \frac{nh}{2\pi} \qquad \text{........ (1)}$$

$$q_1 = e, \quad q_2 = Ze$$

$$\frac{mv^2}{r} = \frac{kq_1q_2}{r^2} = \frac{kze^2}{r^2} \qquad \text{........ (2)}$$

$$\Rightarrow \quad \frac{mn^2h^2}{4\pi^2m^2r^2\cdot r} = \frac{kZe^2}{r^2}$$

$$\Rightarrow \quad r = \frac{mn^2h^2}{4\pi^2m^2kZe^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2mkZe^2}$$

$$\Rightarrow \quad \frac{(6.625\times10^{-34})^2}{4\pi^2\times9.1\times10^{-31}\times9\times10^9\times(1.6\times10^{-19})^2} \times \frac{n^2}{Z}$$

$$r = 0.529 \times \frac{n^2}{z}\,\text{Å}$$

$$r \propto \frac{n^2}{z}$$

for a particular atom $\quad r \propto n^2$

Radius of 1st orbit of H atom $\quad r = 0.529$ Å

Ex. Calculate ratio of radius of 1st orbit of H atom to Li^{+2} ion:

Sol. $\dfrac{\text{Radius of 2}^{\text{nd}}\text{ orbit of H atom}}{\text{Radius of 3}^{\text{rd}}\text{ orbit of Li}^{+2}\text{ atom}} = \dfrac{n^2}{z} \times \dfrac{z_1}{n_1^2} = \dfrac{4}{3} =$

- **Derivation of Velocity of electron in Bohr's orbit:**

$$v = \frac{nh}{2\pi mr}\text{, putting value of r.}$$

$$v = \frac{nh \times 4\pi^2 mkZe^2}{2\pi m^2 h^2}$$

$$v = \frac{2\pi kZe^2}{h.n.} = \frac{2\pi ke^2}{h} \times \frac{Z}{n}$$

$$v = 2.18 \times 10^6 \ \frac{Z}{n} \ m/s$$

$$v \propto \frac{Z}{n}$$

- **Derivation of total energy of electron / system:**

T.E. of system = K.E. of e^- + P.E. of system (nucleus and e^-)

kinetic energy of electron $= \frac{1}{2}\ mv^2 \ = \frac{1}{2}\frac{kze^2}{r}$

$$PE = -\frac{kze^2}{r}$$

$$TE = \frac{-KZe^2}{2r}$$

$$T.E. = -\ 13.6 \times \frac{z^2}{n^2}\ eV/atom$$

$$T.E. = -\ 2.18 \times 10^{-18}\ \frac{z^2}{n^2}\ J/atom$$

As shell no. or distance increases, the value of T.E. and P.E. increases (however magnitude decreases) and becomes maximum at infinity i.e., zero.

negative sign indicates that electron is under the influence of attractive forces of nucleus.

$$K.E. = -\frac{PE}{2}$$

$$T.E. = \frac{P.E}{2}$$

$$T.E = -\ K.E.$$

Calculation of energy of energy level in H atom

(i) When n = 1 (ground level)

K.E. = 13.6 eV (atom)

P.E. = –27.2 eV / atom

T.E. = –13.6 eV/atom

(ii) When n = 2/2nd energy level / 1st excited state

$$K.E. = \frac{13.6}{4} \text{ eV/atom} = 3.4 \text{ eV/ atom}$$

P.E. = – 6.8 eV/atom

T.E. = – 3.4 eV/atom

$E_2 - E_1 = -3.4 + 13.6$ eV/atom = 10.2 eV/atom

(iii) When n = 3 / 3rd energy level/2nd excited state.

$$K.E. = \frac{13.6}{9} \text{ eV/atom} = 1.51 \text{ eV/atom}$$

P.E. = –3.02 eV/atom

T.E. = –1.51 eV/atom

$-E_3 - E_2 = -1.51$ eV/atom + 3.4 eV/atom

= 1.89 eV atom.

(iv) when n = 4 / 3rd excited state

$$K.E. = \frac{13.6}{4^2}$$

$$\Rightarrow \quad = 0.85 \; \frac{13.6}{16\times10} \text{ eV/atom}$$

P.E. = –1.70 eV/atom

T.E. = –0.85 eV/atom

$E_4 - E_3 = 0.66$

As distance increases (n increases) energy of the energy level increases but energy difference between consecutive energy level keeps on increasing i.e., maximum energy difference between 2 to 1 (consecutive)

If reference value (P.E at ∞) is assigned value other than zero. –

(i) All K.E. data remains same.

(ii) P.E./T.E. of each shell will be

changed however difference in P.E./T.E. between 2 shell will remain unchanged.

❖ BOHR'S ATOMIC MODEL

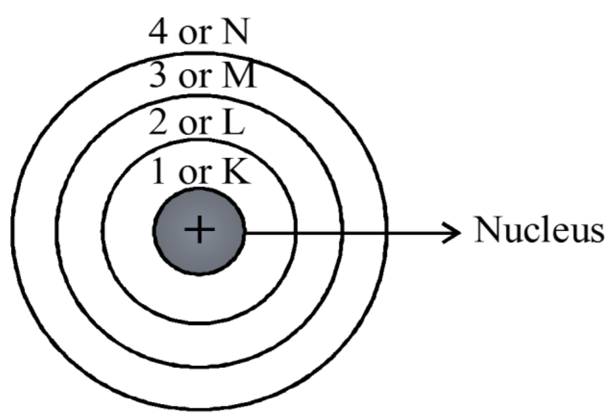
It was the first model based on Planck's quantum theory and the model explained stability of atom and line spectrum of hydrogen.

It is based on quantum theory of light. It is applicable only for single electron species. Ex. H, He^{+}, Li^{2+}, etc.

Assumptions of Bohr's Model

The electron in the hydrogen atom revolves around the nucleus in a circular path of fixed radius and energy. These paths are called orbits, stationary states, energy shells, or allowed energy states. these stationary states for electrons are numbered as n = 1, 2, 3, ... or designated as K, L, M, N, ..., etc. shells (Fig.) These integral numbers

are known as principal quantum numbers. These orbits are arranged concentricallly around the nucleus.

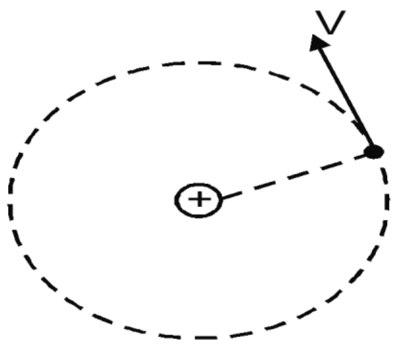


Bohr's orbit

- Electrons revolve only in those orbits where the angular momentum of the electron is quantized. Thus, an electron can move only in those orbits for which its angular momentum is an integral multiple of $h/2\pi$.

$$mvr = n\frac{h}{2\pi}$$

where n = 1, 2, 3, ..., n; h is Planck's constant; m is mass of electron; v is the velocity of electron; and r is the radius of the orbit.



- The energy of an electron in the orbit does not change with time. This means that the energy of an electron in a particular orbit remains constant; it does not lose or gain energy.
- The electron will move from a lower stationary state to a higher stationary state when the required amount of energy is absorbed by the electron. When the electron jumps back to the lower energy level, it emits the same amount of energy. The energy change does not take place in a continuous manner.
- The frequency of radiation absorbed or emitted when transition occurs between two stationary states that differ in energy by ΔE is given by

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

where E_1 and E_2 are the energies of the lower and higher allowed energy states, respectively. This expression is called Bohr's frequency rule.

Mathematical forms of Bohr's Postulates

Calculation of The Radius of The Bohr's Orbit: Suppose that an electron having mass 'm' and charge 'e' revolving around the nucleus of charge 'Ze' (Z is atomic number & e = charge) with a tangential/linear velocity of 'v'. Further consider that 'r' is the radius of the orbit in which electron is revolving.

According to Coulomb's law, the electrostatic force of attraction (F) between the moving electron and nucleus is –

$$F = \frac{KZe^2}{r^2}$$

where $K = \text{constant} = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$

And the centrifugal force $F \frac{mv^2}{r} =$

For the stable orbit of an electron both the forces are balanced.

i.e $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$

then $v^2 = \frac{KKe^2}{mr}$ .... (i)

From the postulate of Bohr,

mvr $\frac{nh}{2\pi} = \Rightarrow \quad v = \frac{nh}{2\pi mr}$

On squaring $v^2 = \frac{n^2h^2}{4\pi^2m^2r^2}$ .... (ii)

From equation (i) and (ii)

$$\frac{KZe^2}{mr} = \frac{n^2 \ h^2}{4\pi^2 \ m^2r^2}$$

On solving, we will get

$$r = \frac{n^2 \ h^2}{4\pi^2 mKZe^2}$$

On putting the value of e , h , m, the radius of n^{th} Bohr orbit is given by :

$$r_n = \frac{n^2}{Z} 0.529 \text{ x } \text{Å} \qquad \propto \frac{n^2}{Z} \Rightarrow$$

Θ $r \Rightarrow \frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \times \frac{Z_2}{Z_1}$

Ex. Calculate radius ratio for 2^{nd} orbit of He^+ ion & 3^{rd} orbit of Be^{+++} ion.

Sol. r_1 (radius of 2^{nd} orbit of He^+ ion) = $\left(\frac{2^2}{2}\right)$ 0.529 Å

r_2 (radius of 3rd orbit of Be^{+++} ion) = $\left(\frac{3^2}{4}\right)$ 0.529 Å

Therefore $$\frac{r_1}{r_2} = \frac{0.529 \times 2^2/2}{0.529 \times 3^2/4} = \frac{8}{9}$$

Calculation of Velocity of an Electron in Bohr's Orbit

Angular momentum of the revolving electron in n^{th} orbit is given by

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \qquad \text{......... (iii)}$$

put the value of 'r' in the equation

then, $$v = \frac{nh \times 4\pi^2 mZe^2 \ K}{2\pi mn^2 \ h^2}$$

$$v = \frac{2\pi Ze^2 K}{nh}$$

on putting the values of π, e^-, h and K

velocity of electron in n^{th} or bit $v_n = 2.18 \times 10^6$ x m/sec ; $v \propto \frac{Z}{n} Z$; $v \propto \frac{1}{n}$

$$v \propto \frac{Z}{n} \Rightarrow \frac{v_1}{v_2} = \frac{Z_1}{Z_2} \times \frac{n_2}{n_1}$$

T, Time period of revolution of an electron in its orbit = $\frac{2\pi r}{v}$ substituting the value of 'r' and 'v' we get

Time Period, $$T = 1.52 \times 10^{-16} \times \frac{n^3}{Z^2}$$

$$\Rightarrow T \propto \frac{n^3}{Z^2}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3} \times \frac{Z_2^2}{Z_1^2}$$

f, Frequency of revolution of an electron in its orbit = $\frac{v}{2\pi r} = \frac{1}{T}$

Calculation of Energy of an Electron

The total energy of an electron revolving in a particular orbit is

$$T.E. = K.E. + P.E.$$

where :

P.E. = Potential energy , K.E. = Kinetic energy , T.E. = Total energy

The K.E. of an electron $= \frac{1}{2} mv^2$

and the P.E. of an electron $= -\frac{KZe^2}{r}$

Hence, T.E. $= \frac{1}{2} mv^2 - \frac{KZe^2}{r}$

we know that, $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$ or $mv^2 = \frac{KZe^2}{r}$ $\Rightarrow$ K.E. $= \frac{1}{2} \frac{KZe^2}{r}$

substituting the value of mv^2 in the above equation:

$$T.E. = \frac{KZe^2}{2r} - \frac{KZe^2}{r} = -\frac{KZe^2}{2r}$$

So, T.E. $= -\frac{KZe^2}{2r}$ $\Rightarrow$ T.E. $= -$ K.E. $= \frac{P.E.}{2}$

substituting the value of 'r' in the equation of T.E.

Then T.E. $= -\frac{KZe^2}{2} \times \frac{4\pi^2 Ze^2 m}{n^2 h^2} = -\frac{2\pi^2 Z^2 e^4 mK^2}{n^2 h^2}$

Thus, the total energy of an electron in n^{th} orbit is given by

$$T.E. = E_n = - \frac{2\pi^2 me^4 k^2}{h^2} \left(\frac{z^2}{n^2}\right) \quad \text{......... (iv)}$$

Putting the value of m,e,h and π we get the expression of total energy

$E_n = -13.6 \frac{Z^2}{n^2}$ eV / atom as the value of n increases , energy of an electron in the orbit increases.

Note: - The P.E. at the infinite = 0

The K.E. at the infinite = 0