WORK AND ENERGY

CONTENTS

➢ WORK

◈ **Definition :** In our daily life "work" implies an activity resulting in muscular or mental exertion. However, in physics the term 'work' is used in a specific sense involves the displacement of a particle or body under the action of a force. "work is said to be done when the point of application of a force moves.

Work done in moving a body is equal to the product of force exerted on the body and the distance moved by the body in the direction of force.

Work = Force × Distance moved in the direction of force.

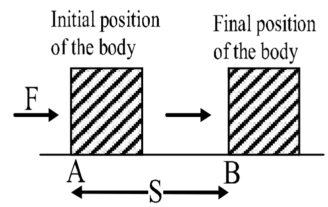
◆ The work done by a force on a body depends on two factors :

(i) Magnitude of the force, and

(ii) Distance through which the body moves (in the direction of force)

◈ **Unit of Work**

When a force of 1 newton moves a body through a distance of 1 metre in its own direction, then the work done is known as 1 joule.



Work = Force × Displacement

1 joule = 1 N × 1 m

or 1 J = 1 Nm (In SI unit)

Ex.1 How much work is done by a force of 10N in moving an object through a distance of 1 m in the direction of the force ?

Sol. The work done is calculated by using the formula:

$$W = F \times S$$

Here, Force, F = 10 N

And, Distance, S = 1 m

So, Work done, W = 10 × 1 J

= 10 J

Thus, the work done is 10 joules

Ex.2 Find the work done by a force of 10 N in moving an object through a distance of 2 m.

Sol. Work done = Force × Distance moved

Here, Force = 10 N

Distance moved = 2 m

Work done, W = 10 N × 2 m

= 20 Joule = 20 J

WORK DONE ANALYSIS

Work done when force and displacement are along same line.

- **Work done by a force :** Work is said to be done by a force if the direction of displacement is the same as the direction of the applied force.
- **Work done against the force :** Work is said to be done against a force if the direction of the displacement is opposite to that of the force.
- **Work done against Gravity :** To lift an object, an applied force has to be equal and opposite to the force of gravity acting on the object. If 'm' is the mass of the object and 'h' is the height through which it is raised, then the upward force

  (F) = force of gravity = mg

  If 'W' stands for work done, then

  W = F . h = mg . h

  Thus W = mgh

  Therefore we can say that, "The amount of work done is equal to the product of weight of the body and the vertical distance through which the body is lifted.

Ex.3 Calculate the work done in pushing a cart, through a distance of 100 m against the force of friction equal to 120 N.

Sol. Force, F = 120 N; Distance, s = 100 m

Using the formula, we have

W = Fs = 120 N × 100 m = 12,000 J

Ex.4 A body of mass 5 kg is displaced through a distance of 4m under an acceleration of 3 m/s^2. Calculate the work done.

Sol. Given :mass, m = 5 kg

acceleration, a = 3 m/s^2

Force acting on the body is given by

F = ma = 5 × 3 = 15 N

Now, work done is given by

W= Fs = 15 N × 4 m = 60 J

Ex.5 Calculate the work done in raising a bucket full of water and weighing 200 kg through a height of 5 m. (Take g = 9.8 ms^{-2}).

Sol. Force of gravity

mg = 200 × 9.8 = 1960.0 N

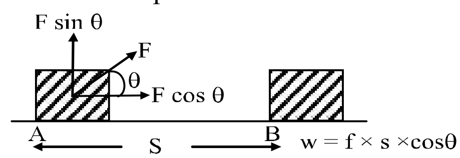
h = 5 m

Work done, W = mgh

or W= 1960 × 5 = 9800 J

Work done when force and displacement are inclined (Oblique case)

Consider a force 'F' acting at angle θ to the direction of displacement 's' as shown in fig.



- **Work done when force is perpendicular to Displacement**

  θ = 90º

  W = F.S × cos 90º = F.S × 0 = 0

  Thus no work is done when a force acts at right angle to the displacement.

Special Examples :

- When a bob attached to a string is whirled along a circular horizontal path, the force acting on the bob acts towards the centre of the circle and is called as the centripetal force. Since the bob is always displaced perpendicular to this force, thus no work is done in this case.
- Earth revolves around the sun. A satellite moves around the earth. In all these cases, the direction of displacement is always perpendicular to the direction of force (centripetal force) and hence no work is done.
- A person walking on a road with a load on his head actually does no work because the weight of the load (force of gravity) acts vertically downwards, while the motion is horizontal that is perpendicular to the direction of force resulting in no work done. Here, one can ask that if no work is done, then why the person gets tired. It is because the person has to do work in moving his muscles or to work against friction and air resistance.

Ex.6 A boy pulls a toy cart with a force of 100 N by a string which makes an angle of 60º with the horizontal so as to move the toy cart by a distance horizontally. Calculate the work done.

Sol. Given F = 100 N, s = 3 m, θ = 60º.

Work done is given by

$$W = Fs\cos\theta = 100 \times 2 \times \cos 60^\circ$$

$$= 100 \times 3 \times \frac{1}{2} = 150\text{ J } (\because \cos 60^\circ = \frac{1}{2})$$

Ex.7 An engine does 64,000 J of work by exerting a force of 8,000 N. Calculate the displacement in the direction of force.

Sol. Given W = 64,000 J; F = 8,000 N

Work done is given by W = Fs

or 64000 = 8000 × s

or s = 8 m

➤ POWER

◈ **Definition :** Power is defined as the rate of doing work

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}} \Rightarrow P = \frac{W}{t}$$

In other words, power is the work done per unit time, power is a scalar quantity.

Since W = F.S therefore

$$P = \frac{W}{t} = \frac{FS}{t} = F \times V = \text{force} \times \text{velocity}$$

◈ **Unit of power :** The S.I. unit of power is watt and it is the rate of doing work at 1 joule per second.

$$1\text{ watt} = \frac{1\text{ joule}}{1\text{ seconds}}$$

1 kilowatt = 1 kW = 1000 W

1 Horse power = 1 H.P. = 746 W

Ex.8 A machine raises a load of 750 N through a height of 15 m in 5s. Calculate :

(i) the work done by the machine.

(ii) the power at which the machine works.

Sol. (i) Work done is given by W = F.s

Here F = 750 N; s = 15 m

∴ W = 750 × 15 = 11250 J

= 11.250 kJ

(ii) Now, power of the machine is given by

$$P = \frac{W}{t}$$

Here, W = 11250 J; t = 5 s

$$\therefore \text{Power } P = \frac{11250\text{J}}{5\text{s}} = 2250\text{ W} = 2.250\text{ kW}$$

Ex.9 A weight lifter lifted a load of 100 kg to a height of 3 m in 10 s. Calculate the following:

(i) amount of work done

(ii) power developed by him

Sol. (i) Work done is given by

W = F . s

Here, F = mg = 100 × 10 = 1000 N

W = 1000 N × 3 m = 3000 joule

(ii) Now, $P = \frac{W}{t}$, where W = 3000 J and t = 10 s

$$\therefore P = \frac{3000\text{ J}}{10\text{s}} = 300\text{ W}$$

Ex.10 A water pump raises 60 liters of water through a height of 20 m in 5 s. Calculate the power of the pump. (Given: g = 10 m/s^2, density of water = 1000 kg/m^3)

Sol. Work done, W = F.s ...(1)

Here, F = mg ...(2)

But,Mass = volume × density

Volume = 60 liters = $60 \times 10^{-3}\ m^3$

Density = 1000 kg/m^3

∴ Mass , m = $(60 \times 10^{-3}\ m^3) \times (1000\ kg/m^3)$

= 60 kg

∴ Equation (2) becomes

F = 60 kg × 10 m/s^2 = 600 N

Now, W = F . s = 600 N × 20 m = 12000 J ∴

$$\text{Power} = \frac{W}{t} = \frac{12000\text{J}}{5\text{s}} = 2400\text{ W}$$

Ex.11 A woman pulls a bucket of water of total mass 5 kg from a well which is 10 m deep in 10 s. Calculate the power used by her.

Sol. Given that m = 5 kg; h = 10 m; t = 10 s

$g = 10\ m/s^2$

$$\text{Now, } P = \frac{W}{t} = \frac{mgh}{t} = \frac{5\times10\times10}{10} = 50W$$

➤ ENERGY

❖ **Definition :** Energy is the ability to do work. The amount of energy possessed by a body is equal to the amount of work it can do when its energy is released. Thus, energy is defined as the capacity of doing work. Energy is a scalar quantity and it exists in various forms.

❖ **Units of energy :** The units of energy are the same as that of work. In SI system, the unit of energy is joule (J). In CGS system, the unit of energy is erg.

$$1 \text{ Joule} = 10^7 \text{ ergs}$$

Other units of energy in common use are watt-hour and kilowatt hour.

$$1 \text{ watt-hour} = 1 \text{ watt} \times 1 \text{ hour}$$
$$= 1 \text{ watt} \times 60 \times 60 \text{ sec}$$
$$= 3600 \text{ J}$$

$$1 \text{ kilowatt-hour (kWh)} = 3.6 \times 10^6 \text{ Joule}$$

Heat energy is usually measured in calorie or kilocalorie such that

$$1 \text{ calorie} = 4.18 \text{ J}$$

A very small unit of energy is electron volt(eV).

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

➤ MECHANICAL ENERGY

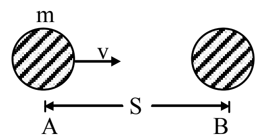
The energy possessed by a body due to its state of rest or state of motion is called mechanical energy.

Mechanical energy is of two types–

(A) Kinetic Energy (B) Potential Energy.

❖ **Kinetic Energy :** The energy of a body due to its motion is called kinetic energy. In other words. The ability of a body to do work by virtue of its motion is called its kinetic energy.

Expression for Kinetic Energy : The kinetic energy of a body is measured in terms of the amount of work done by an opposing force that brings the body to rest from its present state of motion.



Suppose a body of mass m is moving with a velocity v and is brought to rest by an opposing force F.

Now retarding force is given by

$$F = ma \quad ...(1)$$

Now using the equation of motion,

$v^2 - u^2 = 2as$, we get

$$0^2 = v^2 - 2as$$

$$\therefore \quad s = \frac{v^2}{2a} \quad ...(2)$$

Kinetic energy of the body = work done by the retarding force

or Kinetic energy = force × displacement

$$= F . s \quad ...(3)$$

Substituting the value of F from equation (1) and the value of s from equation (2) in equation (3), we get

$$K.E. = ma \times \frac{v^2}{2a} = \frac{1}{2}mv^2$$

Thus, a body of mass m and moving with a velocity v has the capacity of doing work equal to $\frac{1}{2}mv^2$ before it stops.

Ex.12 A bullet of mass 100 gm is fired with a velocity 50 m/s from a gun. Calculate the kinetic energy of the bullet.

Sol. Kinetic energy is given by

$$K.E. = \frac{1}{2}mv^2$$

Here m = 100 gm = 0.1 kg; v = 500 m/s

$$K.E. = \frac{1}{2} \times 0.1 \times (50)^2$$

$$= \frac{1}{2} \times 0.1 \times 50 \times 50 = 125 \text{ J}$$

Ex.13 A 4 kg body is dropped from the top of a building of height 2.5 m. With what velocity will it strike the ground ? What is its kinetic energy when it strikes the ground ?
(Takes $g = 9.8\ m/s^2$)

Sol. Velocity of the body with which it strikes the ground can be calculated by using the equation, $v^2 = u^2 + 2gh$

Here $u = 0$; $g = 9.8\ m/s^2$; $h = 2.5$ m

Substituting these values, we get

$$v^2 = 0^2 + 2 \times 9.8 \times 2.5 = 49$$

or $v = 7$ m/s

Thus, the speed of the body with which it strikes the ground = 7 m/s.

Ex.14 Calculate the velocity of 4 kg mass with kinetic energy of 128 J.

Sol. The formula for kinetic energy is given by

$$K.E. = \frac{1}{2} mv^2$$

Here K.E. = 128 J; m = 4 kg

$$\therefore \quad 128 = \frac{1}{2} \times 4 \times v^2$$

or $v^2 = 64$; or $v = 8$ m/s

Ex.15 Which would have a greater effect on the kinetic energy of an object, doubling the mass or doubling the velocity ?

Sol. (i) The kinetic energy of a body is directly proportional to its "mass" (m). So, if we double the mass (so that it becomes 2m), then the kinetic energy will also get doubled.

(ii) On the other hand, kinetic energy of a body is directly proportional to the "square of its velocity" (v^2). So, if we double the velocity (so that it becomes 2v), then the kinetic energy will become four times. This is because : $(2v)^2 = 4v^2$.

It is clear from the above discussion that doubling the velocity has a greater effect on the kinetic energy of an object.

❖ Potential Energy

Thus the energy possessed by a body by virtue of its position or change in shape is known as potential energy. It is obvious that a body may possess energy even when it is not in motion.

◆ Expression for Potential Energy :

Suppose a body of mass m be lifted from the ground to a vertical height h, then the minimum force required to lift the body is equal to the force of gravity, i.e.

$$F = mg$$

This force of gravity acts on the body vertically downwards.

Now, work done in lifting the body to a height h will be

$$\text{Work} = \text{force} \times \text{distance} = mgh$$

This work done is stored as potential energy in the body such that

Potential energy, U = mgh, i.e. gain in potential energy of the body and the earth.

Ex.16 What will be the potential energy of a body of mass 2 kg kept at a height of 10 m ?

Sol. The potential energy is given by

$$U = mgh$$

Here, m = 2 kg; $g = 10\ m/s^2$; h = 10 m

$$\therefore \quad U = 2 \times 10 \times 10 = 200\ J$$

Ex.17 In lifting a mass of 25 kg to a certain height 1250 J energy is utilized. Calculate to what height it has been lifted ? (Take $g = 10\ m/s^2$)

Sol. In lifting a mass through a height h the work done is given by

$$U = mgh$$

Here, U = 1250 J; $g = 10\ m/s^2$; m = 25 kg

$$\therefore \quad 1250 = 25 \times 10 \times h$$

or $h = 5$ m

➢ LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor be destroyed, it can only be changed from one form to another. Appearing amount of energy in one form is always equal to the disappearing amount of energy in some other form. The total energy thus remains constant.

➢ INTER CONVERSION OF POTENTIAL AND KINETIC ENERGY

Mechanical Energy of a Freely Falling Body :

Assume, a body of mass m is at rest at a height h from the earth's surface, as it starts falling, its velocity after travelling a distance x (point B) becomes v and its velocity on the earth's surface is v'.

Mechanical energy of the body at point A :

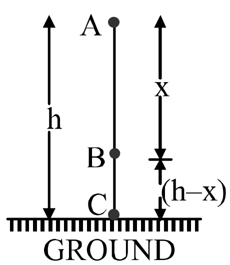
E_A = Kinetic energy + Potential energy

$E_A = m(0)^2 + mgh$

$E_A = mgh$ ……… (i)

Mechanical energy of the body at point B :

$E_B = \frac{1}{2}mv^2 + mg(h-x)$ ……. ..(ii)



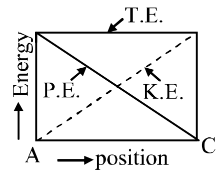
Mechanical energy of the body at point C :

$E_C = \frac{1}{2}m(v')^2 + mg \times 0$

$E_C = \frac{1}{2}m(v')^2$ ……..(iv)

Use : $E_A = E_B = E_c$

Hence, when a body falls freely, its mechanical energy will be constant. That means, the total energy of the body during free fall, remains constant at all positions. However, the form of energy keeps on changing at all points during the motion.



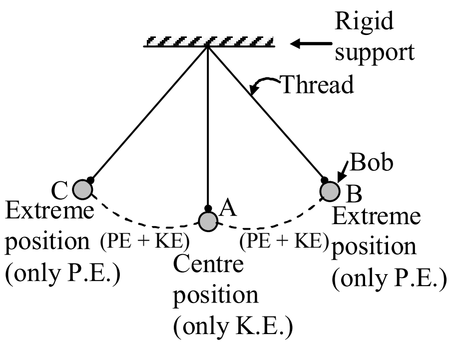
➢ TRANSFORMATION OF ENERGY

- **Definition :** The change of one form of energy into another form of energy is known as transformation of energy.
- **Different Forms of Energy**
  - **Heat energy :** Burning of fuels like diesel or petrol in vehicles provides heat energy to do work.
  - **Electrical energy :** Electric motors are used in home, industry and even for driving electric trains.
  - **Light energy :** When light energy falls on light-meter used in photography, it causes its pointer to move across a scale.
  - **Sound energy :** Sound energy causes a thin plate of microphone diaphragm to vibrate.
  - **Chemical energy :** Chemical energy is the source of energy in our food and it provides us energy to move the various objects.
  - **Nuclear Energy :** The energy in the nucleus of an atom is used to produce heat energy which in turn is used to generate electrical power.

Device used	Energy tranformation	
	Form	to
Steam engine	Heat	Mechanical
Electric fan	Electrical	Mechanical
Electric lamp	Electrical	Light and Heat
Electric heater	Electrical	Heat
Microphone	Sound	Electrical
Solar cell	Solar heat	Electrical
Photo-cell	Light	Electrical
Car engine	Chemical	Heat, Mechanical
Electric cell/batteries	Chemical	Electrical

❖ **A swinging simple pendulum is an example of conservation of energy :**

This is because a swinging simple pendulum is a body whose energy can either be potential or kinetic, or a mixture of potential and kinetic, but its total energy at any instant of time remains the same.



- When the pendulum bob is at position B, it has only potential energy (but no kinetic energy).
- As the bob starts moving down from position B to position A, its potential energy goes on decreasing but its kinetic energy goes on increasing.
- When the bob reaches the centre position A, it has only kinetic energy (but no potential energy).
- As the bob goes from position A towards position C, its kinetic energy goes on decreasing but its potential energy goes on increasing.
- On reaching the extreme position C, the bob stops for a very small instant of time. So at position C, the bob has only potential energy (but no kinetic energy).

Miscellaneous Examples :

Ex.18 A car weighing 1200 kg and travelling at a speed of 20 m/s stops at a distance of 40 m retarding uniformly. Calculate the force exerted by the brakes. Also calculate the work done by the brakes.

Sol. In order to calculate the force applied by the brakes, we first calculate the retardation.

Initial speed, u = 20 m/s; final speed,

v = 0, distance covered, s = 90 m

Using the equation, $v^2 = u^2 + 2as$, we get

$0^2 = (20)^2 + 2 \times a \times 40$

or $80a = -400$

or $a = -5$ m/s^2

Force exerted by the brakes is given by

$F = ma$

Herem = 1200 kg; a = – 5 m/s^2

$\therefore$ $F = 1200 \times (-5) = -6000$ N

The negative sign shows that it is a retarding force. Now, the work done by the brakes is given by

$W = Fs$

Here F = 6000 N; s = 40 m

$\therefore$ $W = 6000 \times 40$ J = 240000 J

$= 2.4 \times 10^5$ J

$\therefore$ Work done by the brakes = 2.4×10^5 J

Ex.19 A horse applying a force of 800 N in pulling a cart with a constant speed of 20 m/s. Calculate the power at which horse is working.

Sol. Power, P is given by force × velocity, i.e.

$P = F . v$

Here F = 800 N; v = 20 m/s

$\therefore$ $P = 800 \times 20 = 16000$ watt

= 16 kW

Ex.20 A boy keeps on his palm a mass of 0.5 kg. He lifts the palm vertically by a distance of 0.5 m. Calculate the amount of work done.

Use $g = 9.8\ m/s^2$.

Sol. Work done, W = F . s

Here, force F of gravity applied to lift the mass, is given by

$$F = mg$$
$$= (0.5\ kg) \times (9.8\ m/s^2)$$
$$= 4.9\ N$$

and $s = 0.5\ m$

Therefore, W = (4.9) . (0.5m) = 2.45 J.

Ex.21 A truck of mass 2500 kg is stopped by a force of 1000 N. It stops at a distance of 320 m. What is the amount of work done ? Is the work done by the force or against the force?

Sol. Here the force, F = 1000 N

Displacement, s = 320 m

∴ Work done, W = F . s

$$= (1000N) . (320\ m)$$
$$= 320000\ J$$

In this case, the force acts opposite to the direction of displacement. So the work is done against the force.

Ex.22 Two bodies of equal masses move with uniform velocity v and 3v respectively. Find the ratio of their kinetic energies.

Sol. In this problem, the masses of the bodies are equal, so let the mass of each body be m. We will now write down the expression for the kinetic energies of both the bodies separately.

(i) Mass of first body = m
Velocity of first body = v

So, K.E. of first body = $\frac{1}{2}mv^2$ ...(1)

(ii) Mass of second body = m
Velocity of second body = 3v

So, K.E. of second body $= \frac{1}{2}m(3v)^2$

$$= \frac{1}{2}m \times 9v^2$$

$$= \frac{9}{2}mv^2 \quad ...(2)$$

Now, to find out the ratio of kinetic energies of the two bodies, we should divide equation (1) by equation (2), so that :

$$\frac{\text{K.E. of first body}}{\text{K.E. of second body}} = \frac{\frac{1}{2}mv^2}{\frac{9}{2}mv^2}$$

or $\frac{\text{K.E. of first body}}{\text{K.E. of second body}} = \frac{1}{9}$ …. (3)

Thus, the ratio of the kinetic energies is 1 : 9.
We can also write down the equation (3) as follows:

K.E. of second body = 9 × K.E. of first body

That is, the kinetic energy of second body is 9 times the kinetic energy of the first body. It is clear from this example that when the velocity (or speed) of a body is "tripled" (from v to 3v), then its kinetic energy becomes "nine times".