DIFFERENTIAL EQUATIONS

7.1 BASIC CONCEPTS

(a) An equation involving derivative(s) of the dependent variable with respect to only one independent variable is called an ***ordinary differential equation***. In particular equation involving $\frac{dy}{dx}$ or dx, dy along with the variables x and y is known as an ordinary differential equation in x and y.

(b) Order of a differential equation is defined as the order of the highest order derivative of dependent variable with respect to the independet variable, involved in the given differential equation.

(c) Degree of a differential equation, when it is polynomial equation in derivatives, is defined as the highest power (exponent) of the highest order derivative involved in the given differential equation.

(d) Solution of a differential equation is a function, in the variables involved in the differential equation, which satisfies the given different equation.

(e) The solution which contains as many arbitrary constants as the order of the differential equation is called the **general solution** (primitive) of the differential equation.

(f) The solution free from arbitrary constants, i.e., the solution obtained from the general solution by giving particular values to arbitrary constants is called a **particular solution** of the differential equation.

7.2 FORMATION OF A DIFFERENTIAL EQUATION

(a) Find how many arbitrary constants are given in the function and how many we are asked to eliminate.

(b) If we have to eliminate one arbitrary constant, then we can differentiate the function once ; if two then we can differentiate twice and so on.

(c) Then from the given function and the expression obtained after differentiation, eliminate the arbitrary constants, to get the differential equation.

7.3 SOLUTION OF DIFFERENTIAL EQUATION

(a) In the solution of a differential equation by **separating the variable method,** take involving x with dx on one side and terms involving y with dy on the other side and integrate both sides. dx and dy should always be in numerator.

(b) Homogeneous differential equation are of the type $\frac{dy}{dx}=\frac{x^2+xy}{y^2}$ or $\frac{dy}{dx}=\frac{x+y}{x-y}$ or $(x^2 + xy)dx - (y^2 + 2xy)dy = 0$, $\frac{dy}{dx}=\frac{x+\sqrt{y^2-x^2}}{x}$ or $\frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right)$, etc, i.e, combined degree of each term involved is same or $\frac{dy}{dx}=f\left(\frac{y}{z}\right)$. We also say that the homogeneous function involved is of degree zero. An equation is a homogeneous equation, if for function involved $f(\lambda x, \lambda y) = f(x, y)$.

In such cases, we substitute $\frac{y}{x}=v$ or $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ and proceed by substituting for $\frac{dy}{dx}$ and y in the given equation. Sometimes, we might get the homogeneous equation as $\frac{dx}{dy}=f\left(\frac{x}{y}\right)$. In such case, we substitute $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ and proceed by substituting for $\frac{dx}{dy}$ and x in the given equation.

(c) In linear differential equation of first order $\frac{dy}{dx}+P(x).\,y = Q(x)$; P(x), Q(x) are function of x.

Find Integrating Factor (I.F.) = $e^{\int Pdx}$ and use the result y. (I.F.) = $\int\{(I.F\}Q(x)\}\,dx$ as solution.

Sometimes, we get linear differnetial equation form as $\frac{dx}{dy}+P(y).x=Q(y);P(y),\,Q(y)$ are funcitons of y.

In such cases find I.F. = $e^{\int Pdy}$ and use the result x. (I.F.) = $\int\{(I.F.)Q(y)\}dy$ as solution.

SOLVED PROBLEMS

Ex.1 ***Write the order and degree of the differential equation*** $\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)=0$

Sol. Highest order derivative is $\frac{dy}{dx}$ $\therefore$ Order of differential equation is 1.

Equation canot be written as a plynomial in derivatives. Hence degre is not defined.

Ex.2 ***What will be the order of the differential equatin, corresponding to the family of curves $y = a \sin (x + b)$, where a is arbitrary constant.***

Sol. As there is one arbitrary constant, so order of corresponding differential equation is 1.

Ex.3 ***Form the different equation representing the family of curves given by $xy = Ae^x + Be^{-x}$ where A and B are constants.***

Sol. $xy = Ae^x + Be^{-x}$ ...(i)

$y + xy' = Ae^x - Be^x$ and $y' + y' + xy'' = xy$ [From(i)] $\Rightarrow xy' + 2y' - xy = 0$ is the required equation

Ex.4 ***Find the equation of a cure passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.***

Sol. We know that the slope of the tangent to a curve at any point (x, y) is given by $\frac{dy}{dx}$.

So, $\frac{dy}{dx}=\frac{2x}{y^2}$ Integrating both sides, we have $\int y^2 dy = \int 2x\,dx$ $\Rightarrow$ $\frac{y^3}{3}=x^2+C$ ...(1)

Since the curve (1) passes through the point (–2, 3) $\frac{(3)^3}{3}=(-2)^2+C$ $\Rightarrow 9 = 4 + C$ $\Rightarrow$ C = 5

Substituting this value of C in (1), we have $\frac{y^3}{3}=x^2+5$ i.e., $y^3 - 3x^2 = 15$ or $y = \sqrt[3]{3x^2+15}$

Ex.5 ***At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the curve, given that it passes through (–2, 1).***

Sol. It is given that $\frac{dy}{dx}=2\left(\frac{-3-y}{-4-x}\right)=2\left(\frac{3+y}{4+x}\right)$ $\Rightarrow \frac{1}{3+y}dy=\frac{2}{4+x}dx$

On integrating, we have, $\Rightarrow \log|3 + y| = 2 \log | 4 + x | + C$

Since the curve passes through the point (–2, 1)

we substitute the value x = –2 and y = 1 in Eq. (1) and get

$\log|3 + 1| = 2 \log|4 - 2| + C$ $\Rightarrow$ $C = \log 4 - 2 \log 2 = 2 \log 2 - 2 \log 2 = 0$

Hence, from (1), the required equation of the curve is

$\log|3 + y| = 2 \log|4 + x|$ or $3 + y = (4 + x)^2$ i.e., $y + 3 = (x + 4)^2$

Ex.6 ***A population grows at the rate of 2% per year. How long does it take for the polulation do double ?***

Sol. Let P_0 be the initial population and let the population after t years be P. Then,

$\frac{dP}{dt} = \frac{2P}{100}$ (given) $\Rightarrow \frac{dP}{P} = \frac{1}{50}dt$ Integrating both sides, we have

$\log P = \frac{1}{50}t + C$ When t = 0, P = P_0 $\therefore$ $\log P_0 = C$ Hence, $\log\left(\frac{P}{P_0}\right) = \frac{1}{50}t$

When P = $2P_0$ $t = 50\log\left(\frac{2P_0}{P_0}\right)$ = 50 log 2 = 50 × 0.3 = 15 years

Thus, it takes 15 years for the population to be double.

Ex.7 ***Show that the differential equation*** $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ ***is homogeneous and solve it.***

Sol. The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)} \quad ...(1)$$

It is a differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$

Here, f(x,y) = $y\cos\left(\frac{y}{x}\right) + x$ and g(x, y) = $x\cos\left(\frac{y}{x}\right)$ Each one is clearly a homogeneous function of degree 1.

Put $y = v x$ or $\frac{y}{x} = v$ ...(2) Differentiating equation (2) with respect to x, we get $\frac{dy}{dx} = v + x\frac{dv}{dx}$ ...(3)

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get $v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$

or $v\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v} - v$ or $x\frac{dv}{dx} = \frac{1}{\cos v}$

...(4)

Separting the variables in equation (4), we get $\cos v\, dv = \frac{dx}{x}$

...(5)

Integrating both sides of equation (5), we get

$\int \cos v\, dv = \int \frac{1}{x}dx$ or sin v = log |x| + log |C| or sin v = log |Cx| ...(6)

Replacing v by $\frac{y}{x}$ in equation (6), we get $\sin\left(\frac{y}{x}\right) = \log|Cx|$ or $y = x\sin^{-1}\{\log|Cx|\}$ ...(7)

which is the general solution of the differential equation (1).

Q.8 ***Show that each of the following differntial equation is homogeneous and solve it :***

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

Sol. The differential equation can be written as

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x} \qquad \Rightarrow f(x, y) \text{ is a homogeneous function of degree 1.}$$

homogeneous function of order 1.

Thus, the given equation is a homogeneous differential equation.

Put y = νx so that $\frac{dy}{dx} = \nu + x\frac{d\nu}{dx}$ Substituting the vaues of y and $\frac{dy}{dx}$ in (1), we have

$$\nu + x\frac{d\nu}{dx} = \frac{\nu x - x\sin\left(\frac{\nu x}{x}\right)}{x} = \nu - \sin\nu \qquad \Rightarrow \qquad x\frac{d\nu}{dx} = -\sin\nu \qquad \Rightarrow \qquad \frac{1}{\sin\nu}.d\nu = -\frac{dx}{x}$$

Integrating both sides, we have

$$\int\frac{1}{\sin\nu}d\nu = -\int\frac{dx}{x} \qquad \Rightarrow \qquad \int \operatorname{cosec}\nu\, d\nu = -\log x + |\log C| \qquad \Rightarrow \qquad \tan\left|\tan\frac{\nu}{2}\right| = -\log x + |\log C|$$

$$\Rightarrow \qquad \tan\frac{\nu}{2} = \frac{C}{x} \qquad \Rightarrow \qquad \tan\frac{y}{2x} = \frac{C}{x} \qquad \text{i.e.,} \qquad x\tan\frac{y}{2x} = C$$

Ex.9 Show that the equation of the curve whose slope at any point (x, y) is equal to y + 2x and which passes through the origin by y + 2(x + 1) = 2e^x

Sol. Here, we have

$$\frac{dy}{dx} = y + 2x \quad \Rightarrow \quad \frac{dy}{dx} - y = 2x \quad \text{Here, P } = -1 \text{ and Q} = 2x \qquad \therefore \ \text{I.F.} = e^{\int P\,dx} = e^{-1\int 1.dx} = e^{-x} = \frac{1}{e^x}$$

So, the general solution of the differential equation (1) is

$$y.\frac{1}{e^x} = \int\frac{2x}{e^x}dx + C \ \Rightarrow \ y.\frac{1}{e^x} = 2\int x.e^{-x}dx + C \ = 2[-xe^{-x} + \int e^{-x}dx] + C \ \Rightarrow \ ye^{-x} = -2xe^{-x} - 2e^{-x} + C$$

Since the curve passes through the origin (0, 0) we have

$$0e^{-0} = -2.0.e^{-0} - 2e^{-0} + C \qquad \Rightarrow \qquad C = 2$$

Hence, the equation of the curve is

$$ye^{-x} = -2xe^{-x} - 2e^{-x} + 2 \qquad \text{i.e.,} \qquad y = -2x - 2 + 2e^x \qquad \text{or} \qquad y + 2(x + 1) = 2e^x$$

Ex.10 Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Sol. According to the question,

$$x + y = \frac{dy}{dx} + 5 \qquad \Rightarrow \qquad \frac{dy}{dx} - y = x - 5$$

Here, P = – 1 and Q = x – 5 ...(1)

$$\text{I.F.} = e^{\int P\,dx} = e^{-\int 1\,dx} = e^{-x}$$

So, the general solution of the D.E. (1) is

$$ye^{-x} = \int (x-5)e^{-x}dx + C \qquad \Rightarrow \qquad ye^{-x} = \int xe^{-x}dx - \int 5e^{-x}dx + C$$

$$\Rightarrow \qquad ye^{-x} = -xe^{-x} - e^{-x} + 5e^{-x} + C \qquad \Rightarrow \qquad y = -x - 1 + 5 + Ce^{x}$$

Since the curve passes thorugh (0, 2), $2 = -0 - 1 + 5 + Ce^{0} \Rightarrow \quad C = -2$

Ex.11 Find the equation of a curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x-coordinate (abscissa) and the product of the x-coordinate and y - coordinate (ordinate) of that point.

Sol. According to the question,

$$\frac{dy}{dx} = x + xy \qquad \Rightarrow \qquad \frac{dy}{dx} - xy = x \qquad \text{............(1)}$$

Here, P = – x and Q = x $\quad \text{I.F.} = e^{\int P\,dx} = e^{\int -x\,dx} = e^{-\frac{x^2}{2}}$

So, the general solution of the D.E. (1) is $ye^{-\frac{x^2}{2}} = \int xe^{-\frac{x^2}{2}}dx + C$ ....(2)

Let $I = \int xe^{-\frac{x^2}{2}}dx$ Put $-\frac{x^2}{2} = t$ so that x dx = – dt $\quad \therefore \quad I = -\int e^{t}dt = -e^{t} = -e^{-\frac{x^2}{2}}$

Hence, from (2) $\quad ye^{-\frac{x^2}{2}} = -e^{-\frac{x^2}{2}} + C \quad$ i.e., $y = -1 + Ce^{\frac{x^2}{2}}$

Since the curve (3) passes through (0, 1), ...(3)

$1 = -1 + Ce^{0} \Rightarrow \quad C = 2 \quad$ So, the required equation of the curve is $y + 1 = 2e^{\frac{x^2}{2}}$

EXERCISE – I UNSOLVED PROBLEMS

Q.1 Determine the order and egree of each of the following differential equation

(i) $x^4\left(\frac{dy}{dx}\right) - 4xy - 3x^3 = 0$ (ii) $\frac{1}{x^2}\left(\frac{d^2y}{dx^2}\right) + 9y = -4e^{-x}$ (iii) $xy\frac{dy}{dx} = \left(\frac{1+y^2}{1+x^2}\right)(1+x+x^2)$

Q.2 Form the differential equation corresponding to $y^2 - 2ay + x^2 = a^2$ by eliminating a.

Q.3 Form the differential equation of the equation $(x + a)^2 - 2y^2 = a^2$ by eliminating a.

Q.4 Form the differential equation corresponding to $(x - a)^2 + 2y^2 = a^2$ by eliminating a.

Q.5 Form the differential equation $y = ax^2 + bx + c$. eliminating a, b, c.

Q.6 Show that $y = A \sin x + B \cos ex + x \sin x$ is a solution of the differential equation $y + \frac{d^2y}{dx^2} = 2e \cos x$

Q.7 Show that $y = \log (x + \sqrt{x^2+a^2})^2$ satisfies the differential equation $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Q.8 Show that $y = ce^{\tan^{-1}x}$ is a solution of the differential equation $(1 + x)^2\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$

Q.9 Show that $y = Ax + \frac{B}{x}$, $x \neq 0$ is a solution of the differential equation $x^2\frac{d^2y}{dx} + x\frac{dy}{dx} - y = 0$

Q.10 Solve the differential equation $\frac{dy}{dx} = \cos^3 x \sin^2 x + x\sqrt{2x+1}$, $x \in \left[-\frac{1}{2}, \infty\right)$

Q.11 Solve the differential equation $\cos x\frac{dy}{dx} - \cos 2x = \cos 3x$

Q.12 Solve the differential equation $(1 + x^2)\frac{dy}{dx} - x = 2\tan^{-1} x$

Q.13 Solve the differential equation $\cos y\, dy + \cos x \sin y\, dx = 0$ given that $y = \frac{\pi}{2}$ when $x = \frac{\pi}{2}$

Q.14 Solve the differential equation $e^{\frac{dy}{dx}} = x + 1$, given that $y = 3$ when $x = 0$.

Q.15 Solve the differential equation $(1 + y^2)\, dx - xy\, dy = 0$, and which passes through (1, 0).

Q.16 Solve the differential equation $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0$, $y(1) = \frac{\pi}{2}$

Q.17 Solve the differential equation $xe^{y/x} - y \sin\frac{\pi}{2} + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$, $y(1) = 0$

Q.18 Solve the differential equation $xy \log\frac{x}{y}dx + \left(y^2 - x^2\log\frac{x}{y}\right)dy = 0$, given that $y(1) = e$

Q.19 Solve the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ $(x \neq 0)$ given that $y(\pi/2) = 0$

Q.20 Solve the differential equation $(1 + y^2)\, dx = (\tan^{-1} y - x)dx$, $y(0) = 0$

Q.21 Solve the differential equation $x\frac{dy}{dx} - y = \log x$, $y(1) = 0$

Q.22 The slope o the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\frac{y}{x}$. Find the equation of the curve.

Q.23 The surface area of a balloon being inflated changes at a rate proportional to time t. If initially its radius is 1 unit and after 1 second it is 3 units, find the radius after t seconds.

Q.24 The popluation grows at the rate of 5% per year. How long does it take four the population to duble ?

Q.25 A radioactive substance disintegrates at a rate proportional to the amount of substance present. If 50% of the given amount disintegrates in 1600 years. What precentage of the substance disintegrates in 10 years ? [Take $e^{-\log 2/160} = 0.9957$]

EXERCISE – II   BOARD PROBLEMS

Q.1 Solve the differential equation $x\frac{dy}{dx}$ = y(log y – log x – 1)

Q.2 Solve the differential equation $\frac{dy}{dx} = 1 - x + y - xy$

Q.3 Solve the differential equation $(x^3 + y^3)dy - x^2ydx = 0$

Q.4 Solve the differential equation $(y + xy)dx + (x - xy^2)\,dy = 0$

Q.5 Show that the differential equation of $y^2 = 4a(x - b)$ is $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Q.6 Solve the differential equation $x\frac{dy}{dx} + y = x^3$ when y(2) = 1

Q.7 Find the differential equation of the family of curves given by $x^2 + y^2 = 2ax$.

Q.8 Solve the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$

Q.9 Form the differential equation of $xy = Ae^x + Be^{-x} + x^2$ where A and B are constant

Q.10 Form the differential equation of curve $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constant.

Q.11 Solve the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$

Q.12 Solve the differential equation $(x^2 - yx^2)\,dy + (y^2 + x^2y^2)\,dx = 0$.

Q.13 Solve the differential equation $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$ when y(e) = e.

Q.14 Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = e^x,\ x > 0$

Q.15 Solve the differential equation $(x^2 + xy)\,dy + (x^2 + y^2)\,dx$

Q.16 Solve the differential equation $(1 + x^2)\frac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$

Q.17 Solve the differential equation $\frac{dy}{dx}$ + secx. y = tan x

Q.18 Solve the differential equation $2xy\,dx + (x^2 + 2y^2)\,dy = 0$

Q.19 Form the differential equation of y = a sin (x + b), where a and b are arbitrary consants.

Q.20 Form the differential equation of y = A cos 2x + B sin 2x when A, B are constant

Q.21 Solve the differential equation $(x\cos y)dy = e^x(x\log x + 1)dx$

Q.22 Solve the differential equation $\frac{dy}{dx}$ + 2y tan x = sin x

Q.23 Solve the differential equation $x^2 \frac{dy}{dx} + 2y \tan x = \sin x$

Q.24 Form the differential equation for the circles passing through origin and with centres on y-axis.

Q.25 Solve the differential equation $x\frac{dy}{dx} = y - x \tan \frac{y}{x}$

Q.26 Solve the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$

Q.27 Solve the differential equation $\frac{dy}{dx} + y = \cos x - \sin x$

Q.28 Solve the differential equation $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

Q.29 Solve the differential equation $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$

Q.30 Solve the differential equation $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

Q.31 Solve the differential equation $(y + 3x^2) \frac{dx}{dy} = x$

Q.32 Form the differential equation of the family of circles in the second. quadrant and touching the coodinate axes.

Q.33 Find the particular solution of the differential equation

$x(x^2 - 1) \frac{dy}{dx} = 1$; $y = 0$ when $x = 2$.

Q.34 Solve the following differential equation :

(`1 + x^2) dy + 2 xy dx = cot x dx; $x \neq 0$

Q.35 Find the particular solution of the differential equation

$(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that when $x = 0$, $y = 0$.

Answers

EXERCISE – 1 (UNSOLVED PROBLEMS)

1. (i) order-1, degree-1 (ii) order-2, degree-1 (iii) order-1, degree-1

2. $(x^2 - 2y^2)\left(\frac{dy}{dx}\right)^2 - 4xy\frac{dy}{dx} - x^2 = 0$ **3.** $x^2 + 2y^2 = 4xy\frac{dy}{dx}$ **4.** $4xy\frac{dy}{dx} = 2y^2 - x^2$

5. $\frac{d^3y}{dx^3} = 0$ **10.** $y = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + C,\ x \in \left[-\frac{1}{2}, \infty\right)$

11. $y = \sin 2x - x + 2\sin x + \log|\sec x + \tan x| + C,\ x \neq (2n + 1)\frac{\pi}{2},\ n \in Z$

12. $y = \frac{1}{2}\log|1 + x^2| + (\tan^{-1} x)^2 + C$ **13.** $\log \sin y + \sin x = 1$ **14.** $y = (x + 1)\log|x + 1| - x + 3$

15. $x^2 = (1 + y^2)$ **16.** $\log|x| = \cos\left(\frac{y}{x}\right)$, $x \neq 0$ **17.** $e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}=1+\log x^2,\ x\neq 0$

18. $\frac{x^2}{2y^2}\log\frac{x}{y} - \frac{x^2}{4y^2}\log y = 1 - \frac{3}{4e^2}$ **19.** $y = x^2 - \frac{\pi^2}{4\sin x}$ $(\sin x \neq 0)$ **20.** $(x - \tan^{-1} y + 1)e^{\tan^{-1}y} = 1$

21. $y = x - 1 - \log x,\ x > 0$ **22.** $y = x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ **23.** $\sqrt{8t^2+1}$

24. $20(\log 2)$ years **25.** 0.43%

EXERCISE – 2 (BOARD PROBLEMS)

1. $y = xe^{cx}$ **2.** $\log(1 + y) = x - \frac{x^2}{2} + c$ **3.** $\frac{-x^3}{3y^3} + \log y = c$ **4.** $\log x + x + \log y - \frac{y^2}{2} = c$

5. $xy = \frac{x^4}{4} - 2$ **7.** $x^2 + 2xy\frac{dy}{dx} - y^2 = 0$ **8.** $y = \tan x - 1 + ce^{-\tan x}$ **9.** $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy - x^2 + 2$

10. $x^2\left(\frac{dy}{dx}-1\right) - 2y^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = 0$ **11.** $y = \cos x + c\cos^2 x$ **12.** $\log y + \frac{1}{y} + \frac{1}{x} - x = c$

13. $y = \frac{2x}{1+\log x}$ **14.** $y = \frac{1}{x}(x - 1)e^x + \frac{c}{x}$ **15.** $\log x + c = -2\log\left(1-\frac{y}{x}\right)-\frac{y}{x}$

16. $y = (x^2 + 1)(x + \tan^{-1} x) + c(x^2 + 1)$ **17.** $y(\sec x + \tan x) = \sec x + \tan x$ **18.** $3x^2y + 2y^3 = c$

19. $\frac{d^2y}{dx^2} + y = 0$ **20.** $\frac{d^2y}{dx^2} + 4y = 0$ **21.** $\sin y = e^x \log x + c$ **22.** $y = \cos x + \cos^2 x$

23. $y = \frac{1}{2}x(x + y)$ **24.** $(x^2 - y^2)\frac{dy}{dx} = 2xy$ **25.** $\log\left(x\sin x\frac{y}{x}\right) = c$ **26.** $y = (\tan x - 1) + ce^{\tan x}$

30. $y + \sqrt{x^2+y^2} = cx^2$ **31.** $y = 3x^2 + cx$ **32.** $(x + y)^2\,|(y')^2 + 1| = (x + yy')^2$

33. $y=\frac{1}{2}\log\left(1-\frac{1}{x^2}\right)-\frac{1}{2}\log\frac{3}{4}$ **34.** $y = (1 + x^2)^{-1}\log|\sin x| + c(1 + x^2)^{-1}$